The Levett School

The Levett School
Positivity | Determination | Reflection | Integrity

Maths Calculations Policy

Policy agreed by Management Committee on:	
Review date for Management Committee:	
Allocated Group/Person to Review:	Helen Megaw
Agreed frequency of Review, by allocated person:	Every Year
Last Review date:	22/05/2025

Melton Road, Sprotbrough, Doncaster, DN5 7SB

Doncaster
Council

Recommended practice delivering a mastery approach

True mastery aims to develop all children’s mathematical understanding at the same pace. As much as possible, children should be accessing the samelearning. Differentiation should primarily be through support, scaffolding and deepening, not through task.

Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, ‘child-friendly’ terminology is used, this must be alongside the current terminology recommended by maths specialists. Using this will support children with their examinations and throughout secondaryschool.

Evidence repeatedly shows that mixed ability seating increases less confident pupils’ perception of mathematical capability, which impacts positively upon outcomes. While not a school policy, it is recommended to avoid ability groups. This presents a challenge in ensuring the more confident mathematicians arebeing extended. An extension tasks to deepen understanding is the most simplistic way around this.

Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to beused with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils.

Used well, manipulatives can enable pupils to inquire themselves- becoming independent learners and thinkers. They can also provide a common language with which to communicate cognitive models for abstract ideas. Drury, H. (2015)

Children aged seven to ten years old work in primarily concrete ways and that the abstract notions of mathematics may only be accessible to them through embodiment in practical resources. Jean Piaget's (1951)

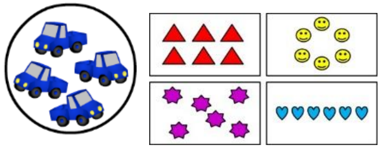
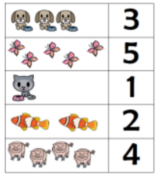
Real things and structured images enable children to understand the abstract. The concrete and the images are a means for children to understand the symbolic so it’s important to move between all modes to allow children to make connections. Morgan, D. (2016)

The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts.

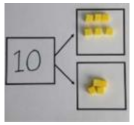
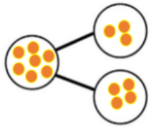
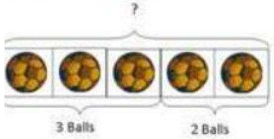
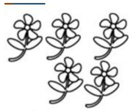
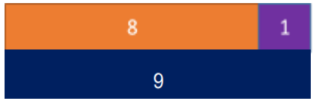
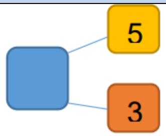
ADDITION

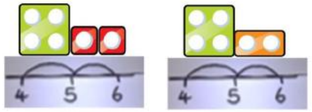
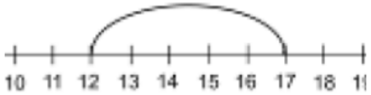
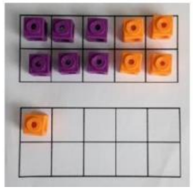
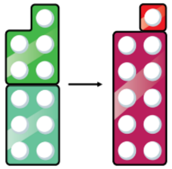
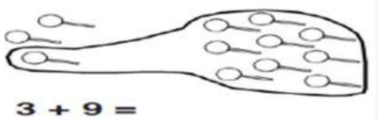
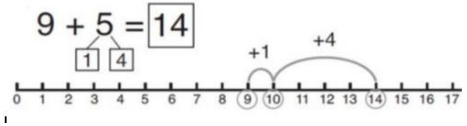
Vocabulary: sum, total, parts and wholes, plus, add, altogether, more, ‘is equal to’, ‘is the same as’

Stage 0 – pre-ARE (EYFS)

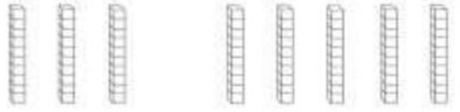
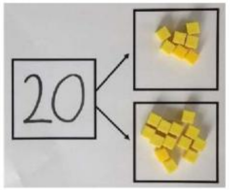
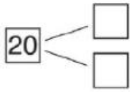
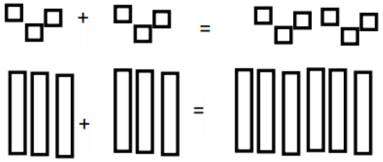
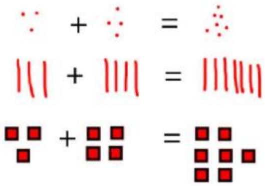
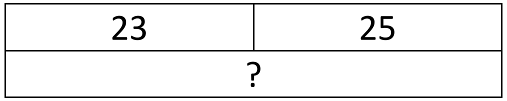
Method	Concrete	Pictorial	Abstract
Counting	Any item of the same things, e.g. pencils, pieces of pasta, shells, counters, cubes, cars, buttons,  	Pictures of the same items in different numbers and laid out differently. 	Relate the number of objects to the numeral. 

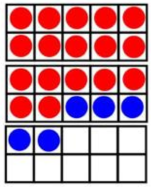
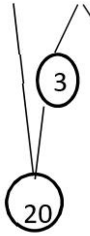
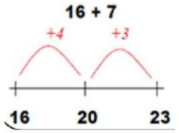
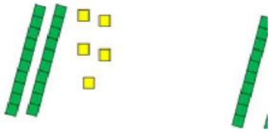
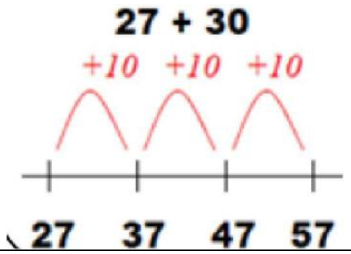
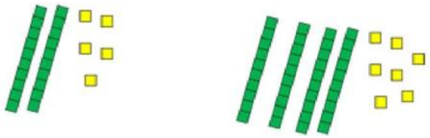
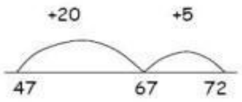
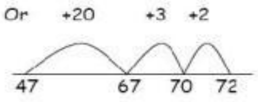
Stage 1 – Year 1

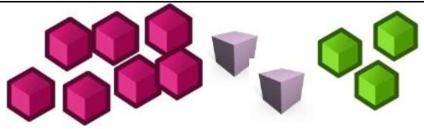
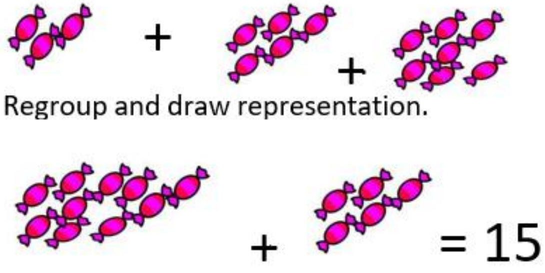
Method	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part whole model	  For 4 + 3, count out 4 items then 3 more and group them together to see what you have altogether. Represent in a bar / in a group   Other resources can be used instead of cubes (teddy’s, cars, shells, counters, etc).	Use pictures to add two numbers together as a group or in a bar.     	 Use the part – part – whole diagram as shown above to move into the abstract. 8 = 5 + 3 5 + 3 = 8 Include missing number questions to support varied fluency: 8 = ? + 3 5 + ? = 8

Starting at the bigger number and count on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.  Using number lines using cubes/ Numicon   	12 + 5 = 17  Start at the larger number from the sum on the number line and count on in 1s or jump to find the answer.	5 + 12 = 17 Place the number line in your head and count on the smaller number to find your answer. Variation of questions. With the number line in your head: • What is 2 more than 4? • What is the sum of 4 and 4? • What's the total of 4 and 2? • 4+2
'The Magic 10' **Regrouping to make 10** Makes the calculation easier. Essential for column addition later.	Regrouping 9 +3 into 10 + 2 before adding together  Start with the bigger number and use the smaller number to make 10 using ten frames or numicon: 6 + 5 = 11  	Children to draw the ten frame and counters/cubes. Use pictures or a number line. Regroup to partition the smaller number using the part – part- whole model to make 10.   	7 + 5 = ? 7 + 3 + 2 = ? If I have 7 how many of my 5 do I need to make 10? How many more do I still need to add on? Children to develop an understanding of equality 6 + ☐ = 11 6 + 5 = 5 + ☐ 6 + 5 = ☐+ 4

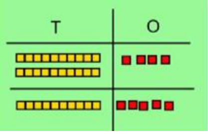
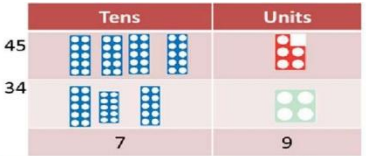
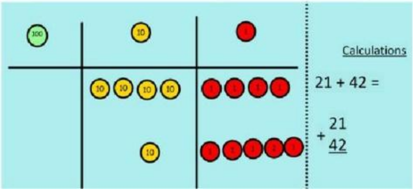
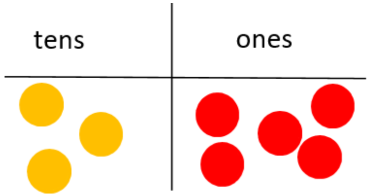
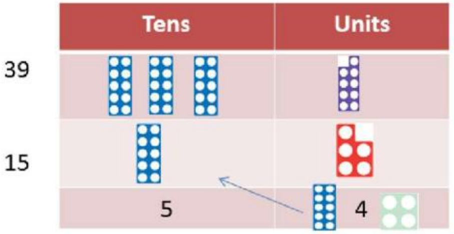
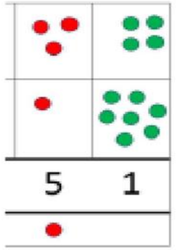
Stage 2 – Year 2

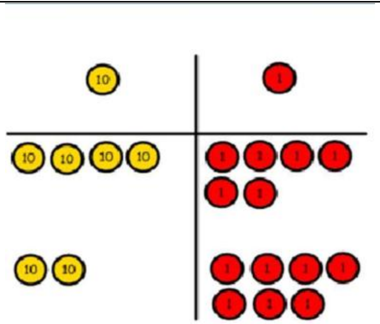
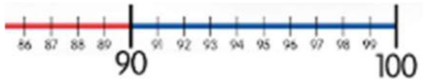
Method	Concrete	Pictorial	Abstract
Adding multiples of 10	50 = 30 + 20  Model using dienes and beadstrings.	Use representations of base ten.  3 tens + 5 tens = ______ tens 30 + 50 = ______	20 + 30 = 50 70 = 50 + 20 40 + ☐ = 60 Ensure all variations of sums layout is done.
Use known number facts Part, part whole	 Children explore ways of making numbers within 20.	 ☐ + ☐ = 20 20 – ☐ = ☐ ☐ + ☐ = 20 20 – ☐ = ☐	Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations. ☐ + 1 = 16 16 – 1 = ☐ 1 + ☐ = 16 16 – ☐ = 1
Using known facts		 Children draw representations of H, T and O.	3 + 4 = 7 leads to 30 + 40 = 70 leads to 300 + 400 = 700
Bar Model	 3 + 4 = 7	 7 + 3 = 10	

Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern.17 + 5 = 22 27 + 5 = 32	17 + 5 = 22 Use part part whole and number line to model.  	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17  Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.
Add a two digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change		27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2 digit numbers	 Model using dienes , place value counters and numicon	Use number line and bridge ten using part whole if necessary.  	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 =12 60 + 12 = 72 Lead into recording in column format, to reinforce place value and prepare childrenfor formal written methods with larger values.

Add three 1 digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation. 	$\underbrace{4 + 7 + 6}_{10} = \boxed{10} + \boxed{7}$ $= \boxed{17}$ Combine the two numbers that make/bridge ten then add on the third.

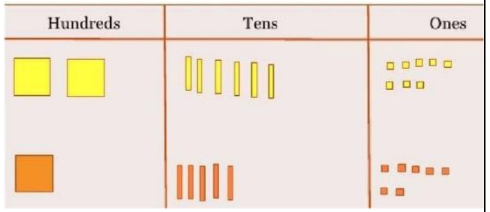
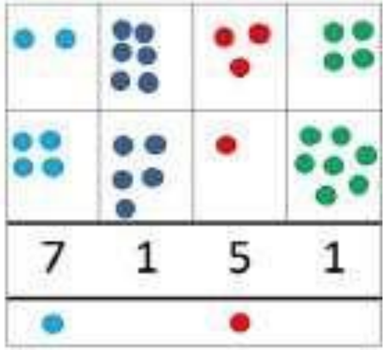
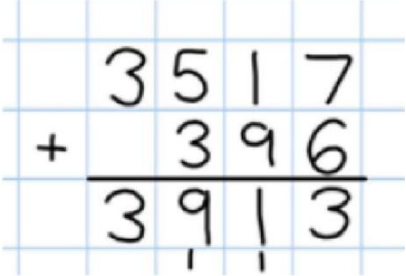
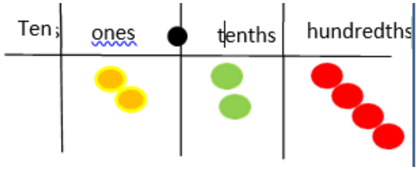
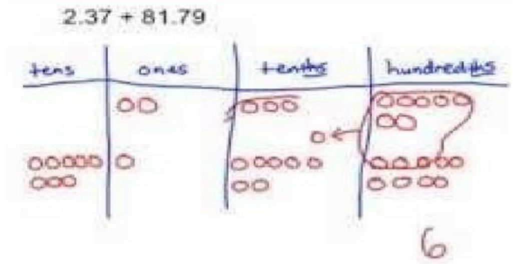
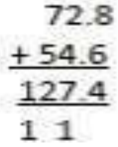
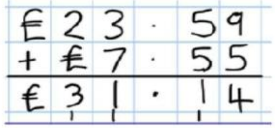
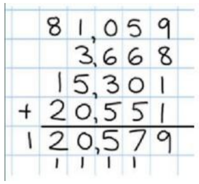
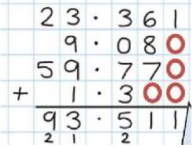
Stage 3 – Year 3

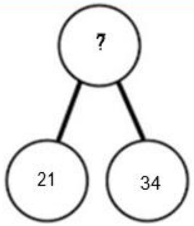
Method	Concrete	Pictorial	Abstract
Column Addition— no regrouping (friendly numbers) Add two or three 2 or3digit numbers.	 Dienes or numicon Add together the ones first, then thetens.   Move to using place value counters	Children move to drawing the counters using atens and one frame. 	$\begin{array}{r} 2\ 2\ 3 \\ +\ 1\ 1\ 4 \\ \hline 3\ 3\ 7 \end{array}$ Add the ones first, then the tens, thenthe hundreds.
Column Addition with regrouping .	 Exchange ten ones for a ten. Model using numicon and place value counters.	 $\begin{array}{r} 3\ 4 \\ +1\ 7 \end{array}$ Children can draw a representation of the grid to further support their understanding,carrying the ten **underneath** the line	$\begin{array}{rcrl} 20 & + & 5 & \\ 40 & + & 8 & \\ \hline 60 & + & 13 & = 73 \end{array}$ Start by partitioning the numbers before formal column to showthe exchange. $\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ \hline 1\,1 \end{array}$

	 46 + 27 = 73		
Estimate the answers to questions and use inverse operations to check answers	 Estimating 98 + 17 = ? 100 + 20 = 120	Use number lines to illustrate estimation. 	Building up known facts and using them to illustrate the inverse and to check answers. 98 + 18 = 116 116 – 18 = 98 18 + 98 = 116 116 – 98 = 18

Stage 4 – Year 4-6

Method	Concrete	Pictorial	Abstract
Years 4 – 6 Estimate and use inverse operations to check answers to a calculation	**As per Year 3**		

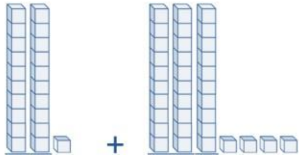
Y4—add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundredand ten hundreds for a thousand.  	 Draw representations using place value grid.	 Continue from previous work to carryhundreds as well as tens. Relate to money and measures.
Y5—add numbers with	As year 4  Introduce decimal place value countersand model exchange for addition.	 	 
Y6—add several numbers of increasing complexity, inc adding money, measureand decimals with different numbers of decimal points.	As Y5	As Y5	Insert zeros for place holders.  
Conceptual variation; different ways to ask pupils to solve 21+34			

<table>
<tr>
<td>


<table>
<tr><td colspan="2">?</td></tr>
<tr><td>21</td><td>34</td></tr>
</table>
</td>
<td>
Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

21 + 34 = 55. Prove it
</td>
<td>
$$\begin{array}{r} 21 \\ \underline{+34} \\ \hline \end{array}$$
21 + 34 =

$\boxed{}$ = 21 + 34

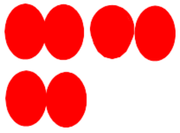
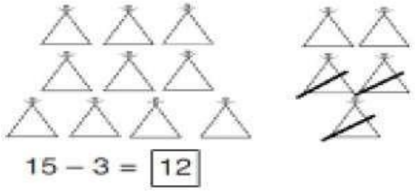
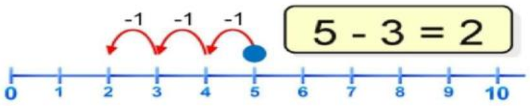
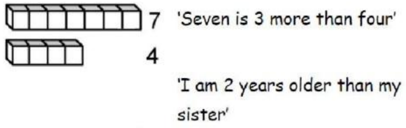
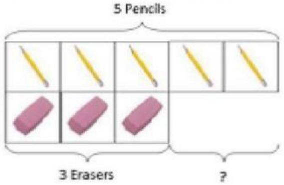
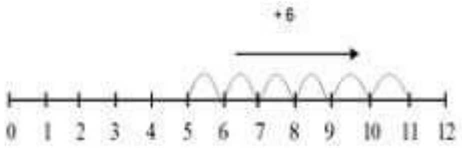
Calculate the sum of twenty-one and thirty-four.
</td>
<td>


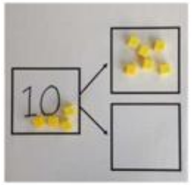
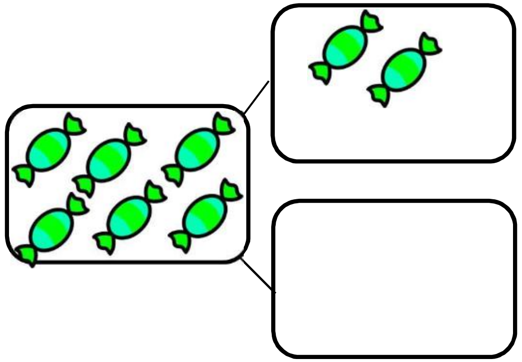
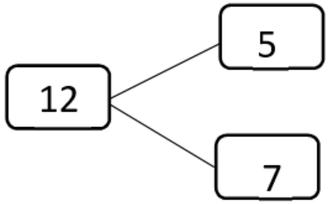
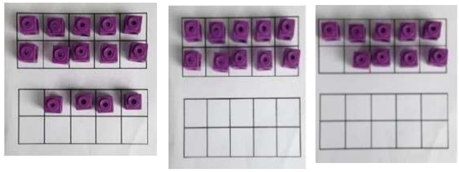
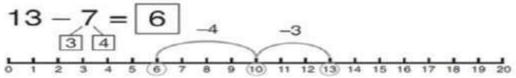
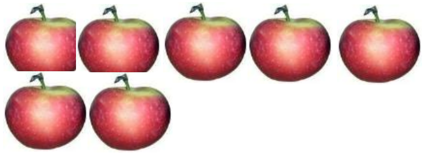
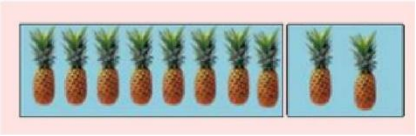
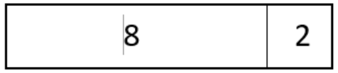
Missing digit problems:
<table>
<tr><th>10s</th><th>1s</th></tr>
<tr><td>10 10</td><td>1</td></tr>
<tr><td>10 10 10</td><td>?</td></tr>
<tr><td>?</td><td>5</td></tr>
</table>
</td>
</tr>
</table>

SUBTRACTION

Vocabulary: take away, less than, the difference, subtract, minus, fewer, decrease

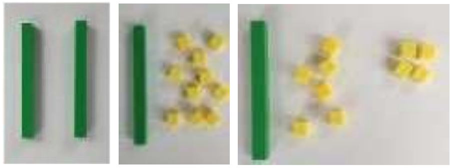
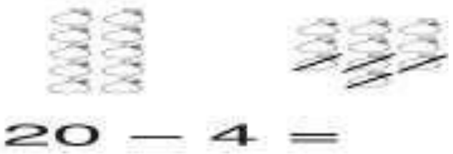
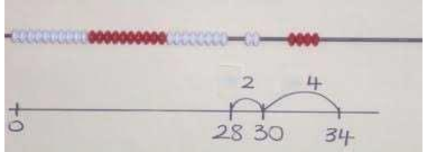
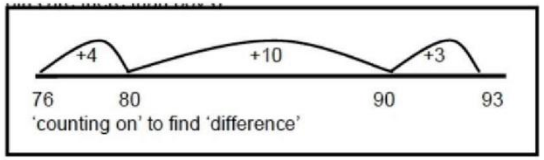
Stage 1 – Year 1

Method	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etcto show how objects can be taken away. 4—2 = 2  6—4 = 2 	Cross out drawn objects to show what hasbeen taken away. 	7—4 = 3 16—9 = 7
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you countbackwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. Whatnumber are you at?
Find the Difference	Compare objects and amounts  	Count on using a number line to find the difference. 	Hannah has12 sweets and her sister has 5. How many more does Hannah have than hersister.?

	Lay objects to represent bar model.		
Represent and use number bonds and related subtraction facts within 20 Include subtracting zero Part Part Whole model	 Link to addition. Use PPW model to modelthe inverse. If 10 is the whole and 6 is one of the arts,what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within thepart whole model.  Include missing number problems: 12 - ? = 5 7 = 12 - ?
Make 10 using a ten frame	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away sothat you have taken 5.	 13—7 Jump back 3 first, then another 4. Use tenas the stopping point.	16—8 How many do we take off first to get to 10?How many left to take off?
Bar model Including the inverse operations.	 5—2 = 3		 10 = 8 + 2 10 = 2 + 8 10—2 = 8

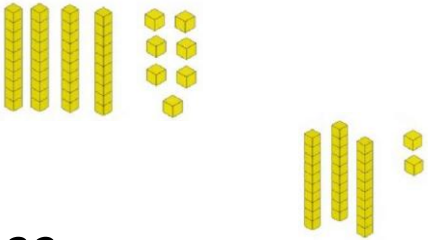
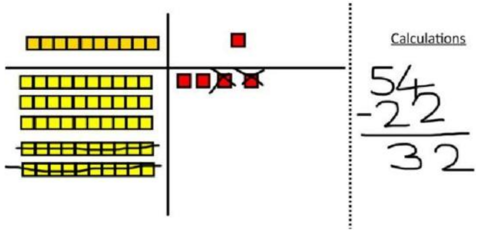
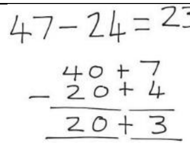
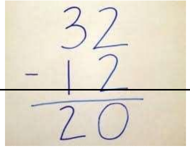
			10—8 = 2

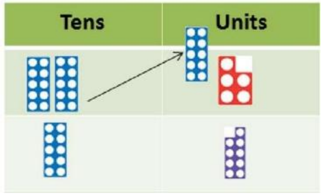
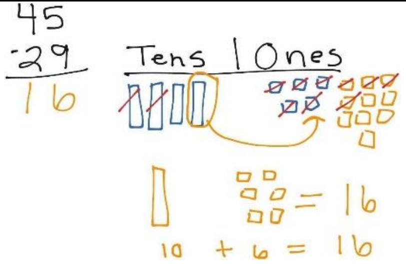
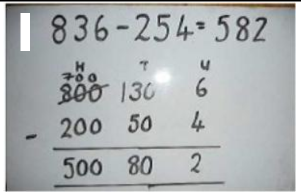
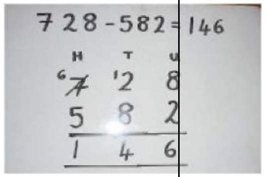
Stage 2 – Year 2

Method	Concrete	Pictorial	Abstract
Regroup a ten intoten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take andmake'		20—4 = 16
Partitioning to subtract without regrouping. *'Friendly numbers'*	34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes andcross off.  43—21 = 22	43—21 = 22
Make ten strategy *Progression should be crossing one ten, crossingmore than one ten,*	 34—28	 Use a number line to count on to next tenand	93—76 = 17

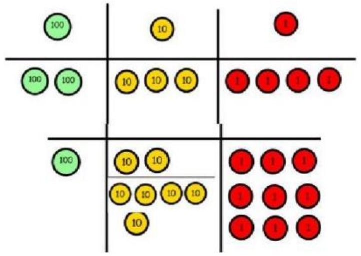
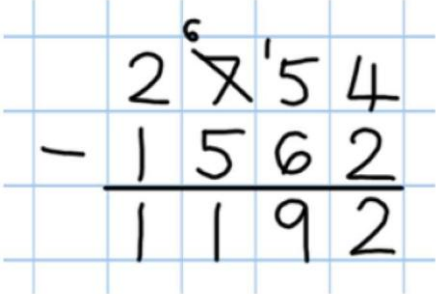
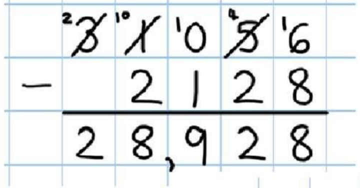
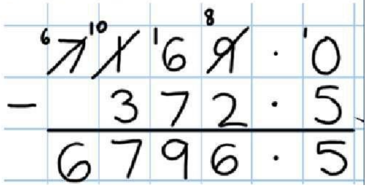
crossing the hundreds.	Use a bead bar or bead strings to modelcounting to next ten and the rest.	then the rest.	

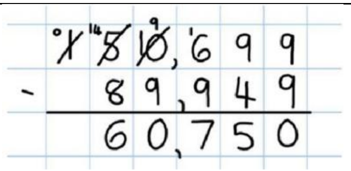
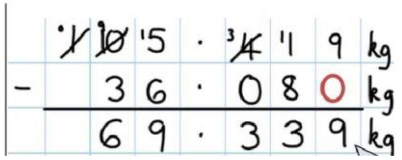
Stage 3 – Year 3

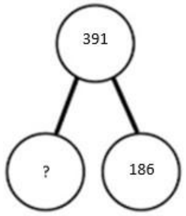
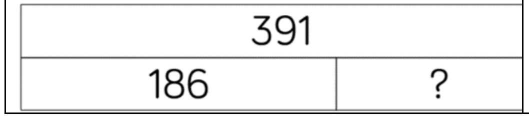
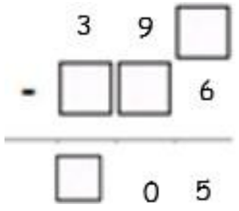
Method	Concrete	Pictorial	Abstract
Subtract numbers mentally, including: three digit number + ones three digit number + tens three digit number + hundreds			Vary the position of the answer and question. Expose children to missing number questions and vary the missing part of the calculation. 678 = ? - 1 688 – 10 = ? 678 = ? – 100
Column subtraction without regrouping (friendly numbers)	 47—32 Use base 10 or Numicon to model	 Draw representations to supportunderstanding	 $47-24=23$ $40+7$ $-\ 20+4$ $20+3$ Intermediate step maybe needed to lead to clear subtraction understanding.  32 - 12 20

Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take andmake' for exchange.	 Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns Then move to formal method. 

Stage 4 – Years 4-6

Method	Concrete	Pictorial	Abstract
Subtracting tens andones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon,base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with*	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for placeholder 

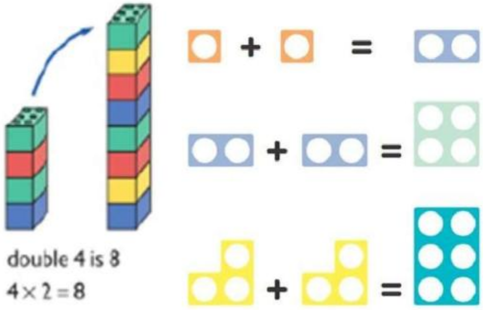
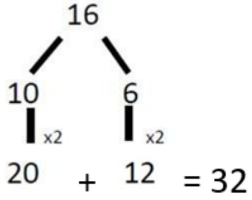
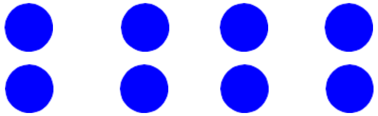
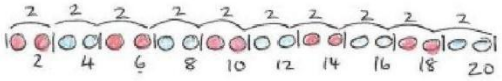
decimal values, including mixturesof integers and decimals and aligning the decimal *Up to 3 decimal places*			
Year 6— Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).	As Year 4	Children to draw pv counters and show their exchange—see Y3	 

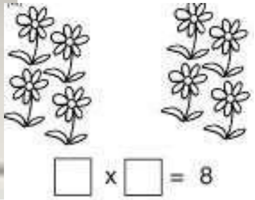
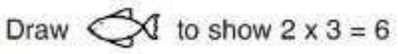
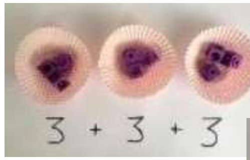
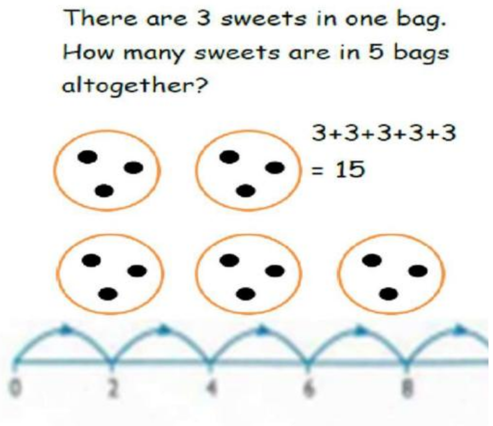
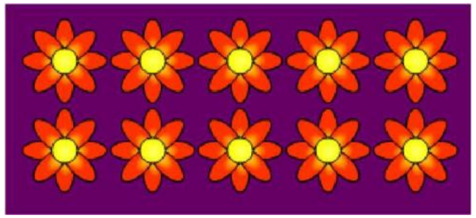
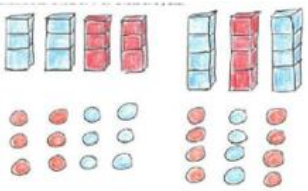
Conceptual variation; different ways to ask pupils to solve 391- 186			
 	Raj spent £391, Timmy spent £186. How much more did Raj spend? Calculate the difference between 391 and 186.	☐ = 391 – 186 391 -186 ____ What is 186 less than 391?	Missing digit calculations 

MULTIPLICATION

Vocabulary: double, times, multiplied by, the product of, groups of, lots of, equal groups

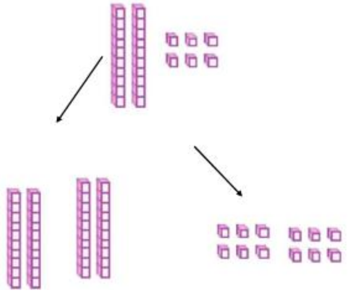
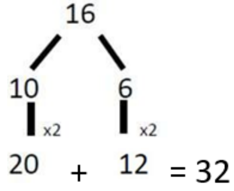
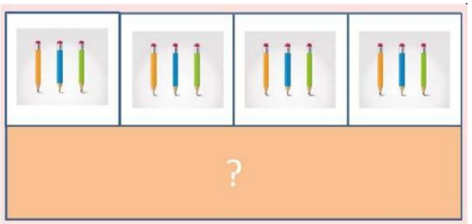
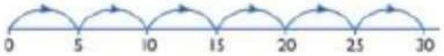
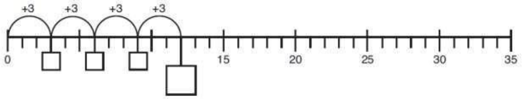
Stage 1 – Year 1

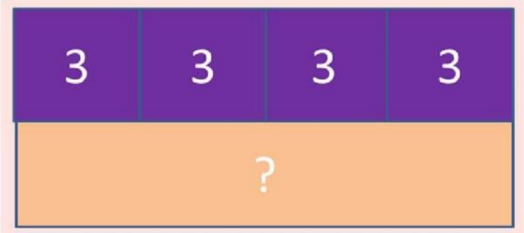
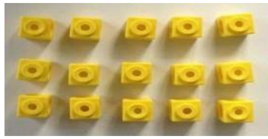
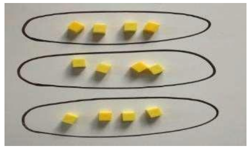
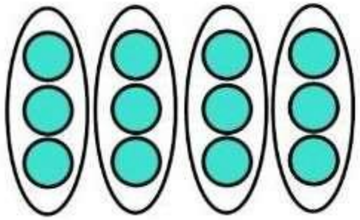
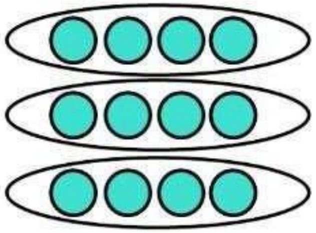
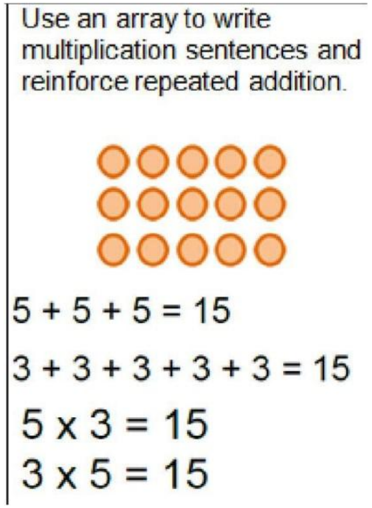
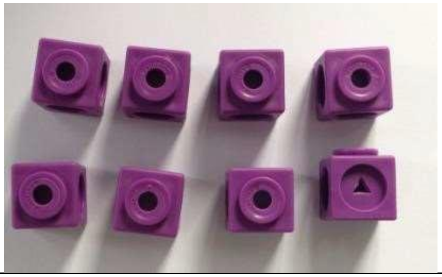
Method	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling  	Draw pictures to show how to double numbers Double 4 is 8 	Partition a number and then double each partbefore recombining it back together. 
Counting in multiples (2s, 5s, 10s)	Count the groups as children are skip counting, children may use their fingers as they are skip counting.  	 Children make representations to show counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples ofnumbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

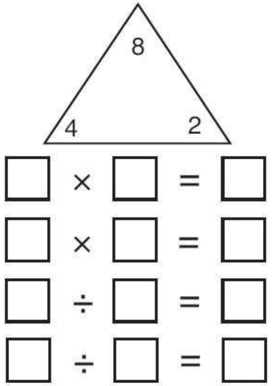
Making equal groups and counting the total	  Use manipulatives to create equal groups.	 Draw to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8
Repeated addition	   Use different objects to addequal groups	Use pictorial including number lines to solve problems  There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 	Write addition sentences to describe objectsand pictures. 
Understanding arrays	Use objects laid out in arrays to find theanswers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show  Understanding,	3 x 2 = 6 2 x 5 = 10

Stage 2 – Year 2

Children should be able to recall and sue multiplication and division facts for the 2, 5 and 10 times tables.

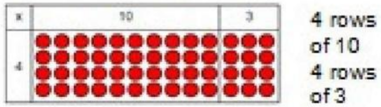
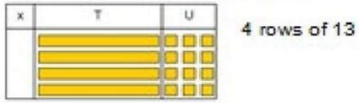
Method	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes andPV counters.  40 + 12 = 52	Draw pictures and representations toshow how to double numbers	Partition a number and then double eachpart before recombining it back together. 
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children areskip counting, children may use their fingers as they are skip counting. Use bar models.   5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40 	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   	Count in multiples of a number aloud. Write sequences with multiples ofnumbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 $4 \times 3 = \square$

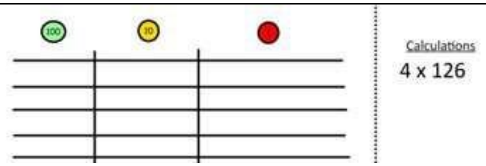
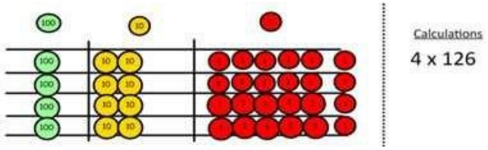
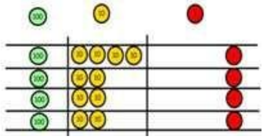
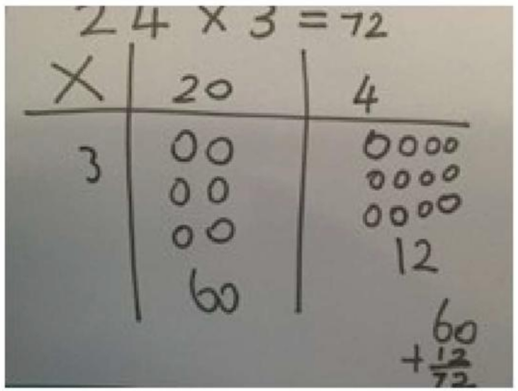
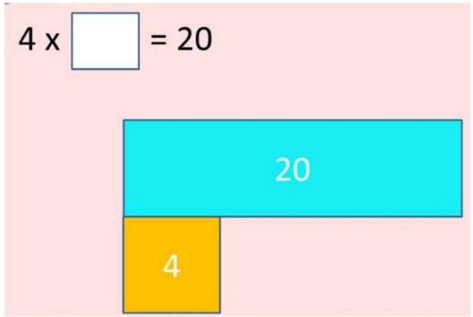
			
Multiplication is commutative	Create arrays usingcounters and cubes and Numicon.  Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. 	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3  Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4

			4 = 8÷ 2 Show all 8 related fact family sentences.

Stage 3 – Year 3

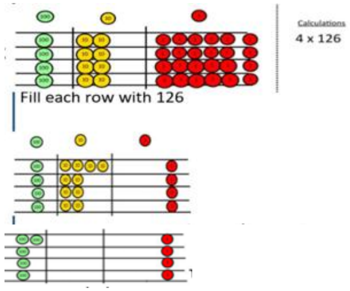
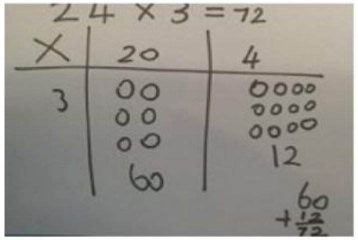
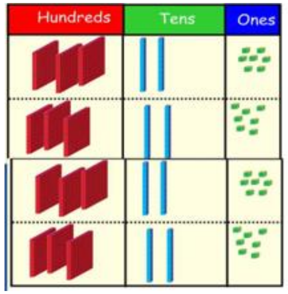
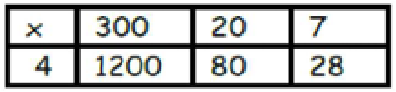
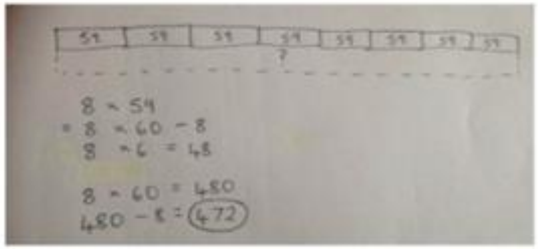
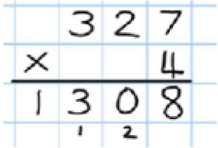
Children should be able to recall and sue multiplication and division facts for the 2, 5 and 10 times tables.

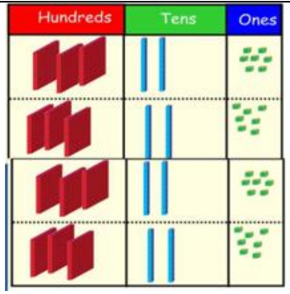
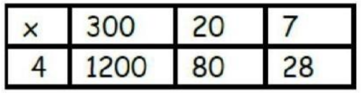
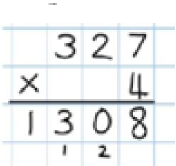
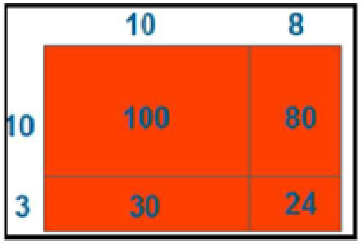
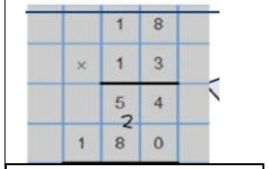
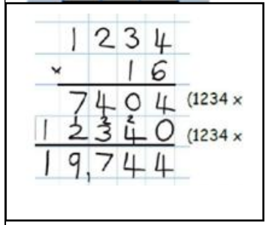
Method	Concrete	Pictorial	Abstract
Grid method, progressing to theformal method Multiply 2 digit numbers by 1 digit numbers	Show the links with arrays to first introduce the grid method.  Move onto base ten to move towards a morecompact method.  Move on to place value counters to show how we are finding groups of a number. We aremultiplying by 4 so we need 4 rows	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles inthe different columns to show their thinking as shown below.	Start with multiplying by one digit numbers and showing the clear addition alongside thegrid.  × 30 5 7 210 35 210 + 35 = 245 Move forward to the formal written method: 3 5 X 7 2 4 5

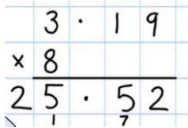
	  Fill each row with 126. Add up each column, starting with the onesmaking any exchanges needed Then you have your answer. 	 	3
Solve problems, including missing number problems, integer scaling problems,			Three times as high, eight times as long ? x 5 = 20 20 ÷ ? = 5 3 hats and 4 coats, how many differentoutfits?

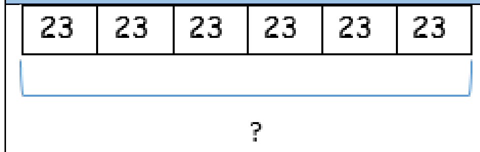
Stage 4 – Years 4-6

Method	Concrete	Pictorial	Abstract
Grid method recap from Year 3 for 2 digits x 1 digit	Use place counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show	Start by multiplying by one digit numbers and showing the clear addition alongside the grid.

Move to multiplying 3 digit numbers by 1 digit. (Year 4 expectation).	 Add up each column making any exchanges as needed.	different amounts or just use the circles in the different columns to show their thinking as shown below. 	x 30 5 7 210 35 210 + 35 = 245
Column multiplication	Children can continue to be supported by place value counters at this stage of multiplication. This is initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside.	x 300 20 7 4 1200 80 28 The grid method may be used to show how this relates to a formal written method.  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	327 x 4 28 80 1200 1308 This may lead to a compact method. 

Column Multiplication for3 and 4 digits x 1 digit.	 It is important at this stage that they always Multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642	 x | 300 | 20 | 7 4 | 1200 | 80 | 28	327 x 4 28 80 1200 1308 
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	 Continue to use bar modelling to support problem solving	  18 x 3 on thefirst row (8 x 3 =24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first
Multiplying decimalsup to 2 decimal places by a single digit.			Remind children that the single digit belongsin the units column. Line up the decimal points in the question and the answer.

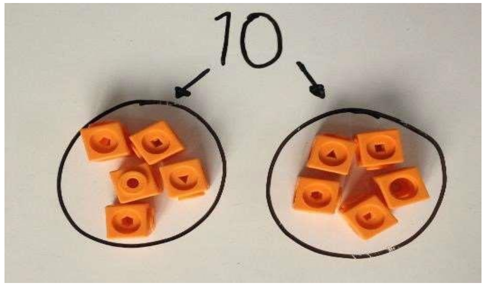
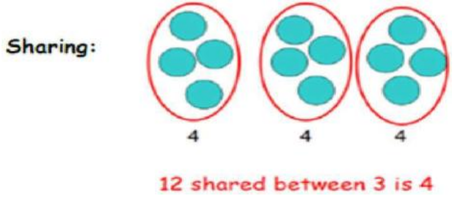
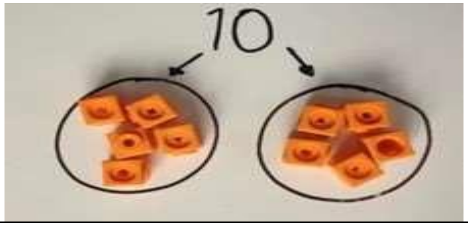
Conceptual variation; different ways to ask pupils to solve			
	Mai had to swim 23 lengths, 6 timesa week. How many lengths did she swim inone week? With the counters, prove that 6 x 23 = 138	Find the product of 6 and 23 6 × 23 = ☐ = 6 × 23 6 23 × 23 × 6	What is the calculation?What is the product? 

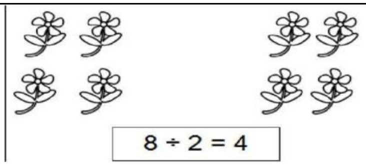
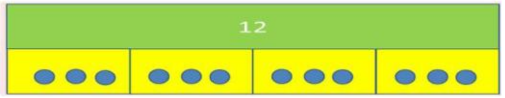
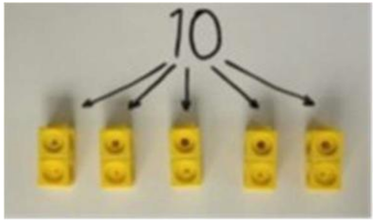
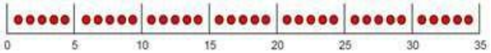
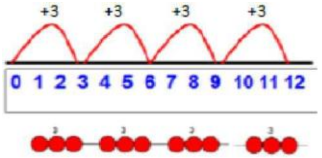
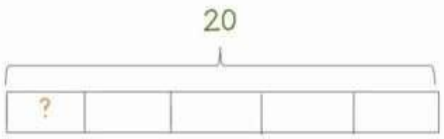
DIVISION

Vocabulary: share, group, divide, divided by, half

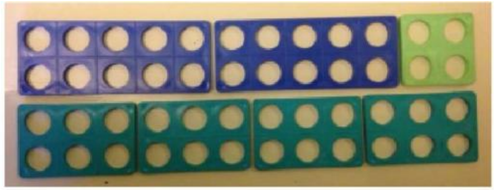
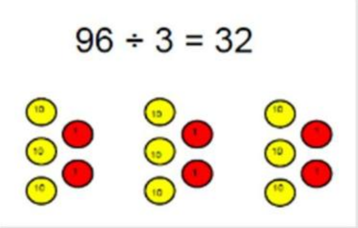
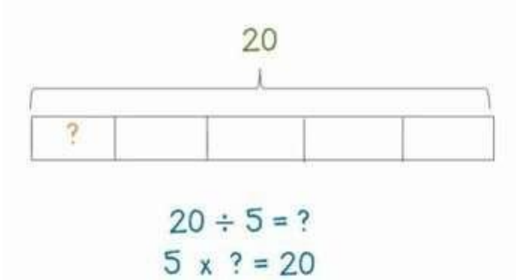
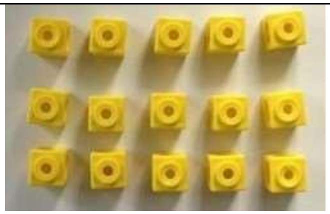
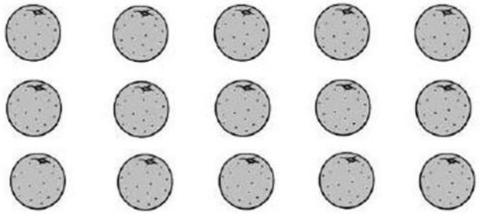
Stage 1 – Year 1

Method	Concrete	Pictorial	Abstract
Division as sharing *Use Gordon ITPs for modelling*		Children use pictures or shapes to share quantities.  8 shared between 2 is 4	12 shared between 3 is 4

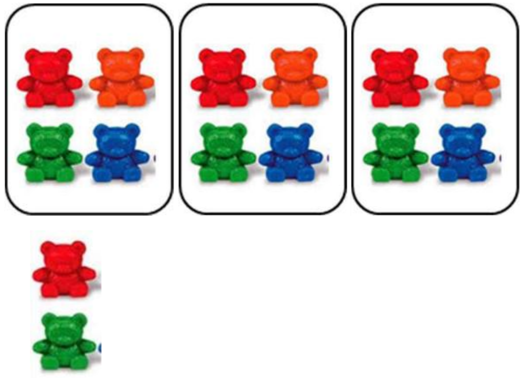
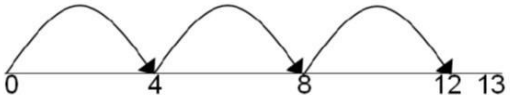
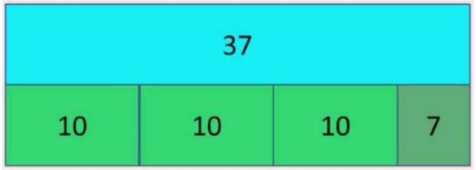
	   I have 10 cubes, can you share them equally in2 groups?		
Division as sharing		Children use pictures or shapes to share quantities.	12 ÷ 3 = 4

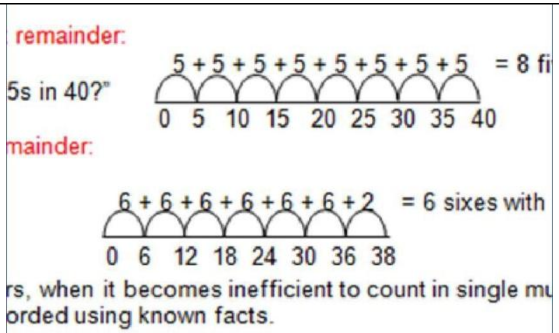
	I have 10 cubes, can you share them equally in2 groups?	 Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  20 ÷ 5 = ? 5 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are ineach group?

Stage 2 – Year 2

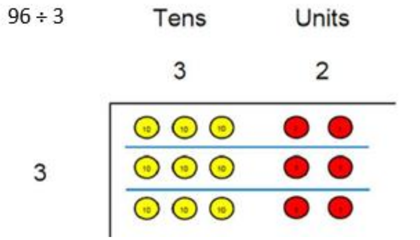
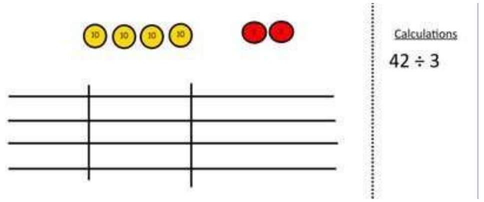
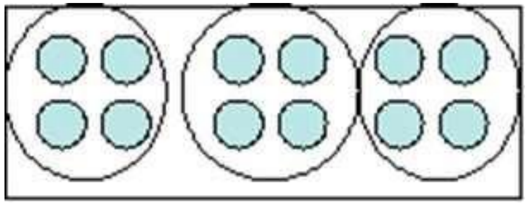
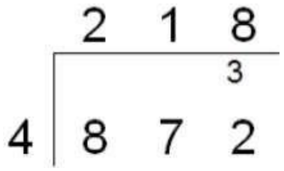
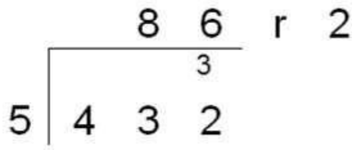
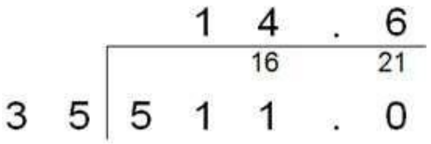
Method	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4  96 ÷ 3 = 32 	Continue to use bar modelling to aid solving division problems.  20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 =15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

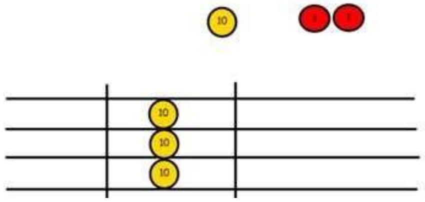
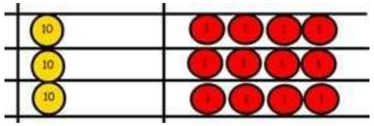
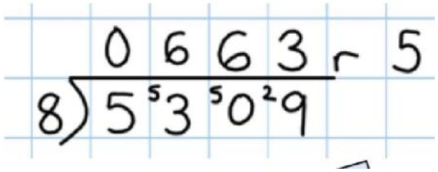
Stage 3 – Year 3 (Greater Depth Year 2)

Method	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups andsee how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders. 	Complete written divisions and show theremainder using r. 

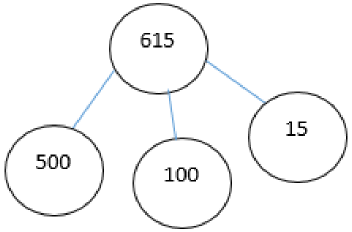
		 rs, when it becomes inefficient to count in single mu orded using known facts.	

Stage 4 – Year 4-6

Method	Concrete	Pictorial	Abstract
Divide at least 3 digitnumbers by 1 digit. Short Division	  Use place value counters to divide using the bus stop method alongside 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally withno remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide thetotal accurately. 

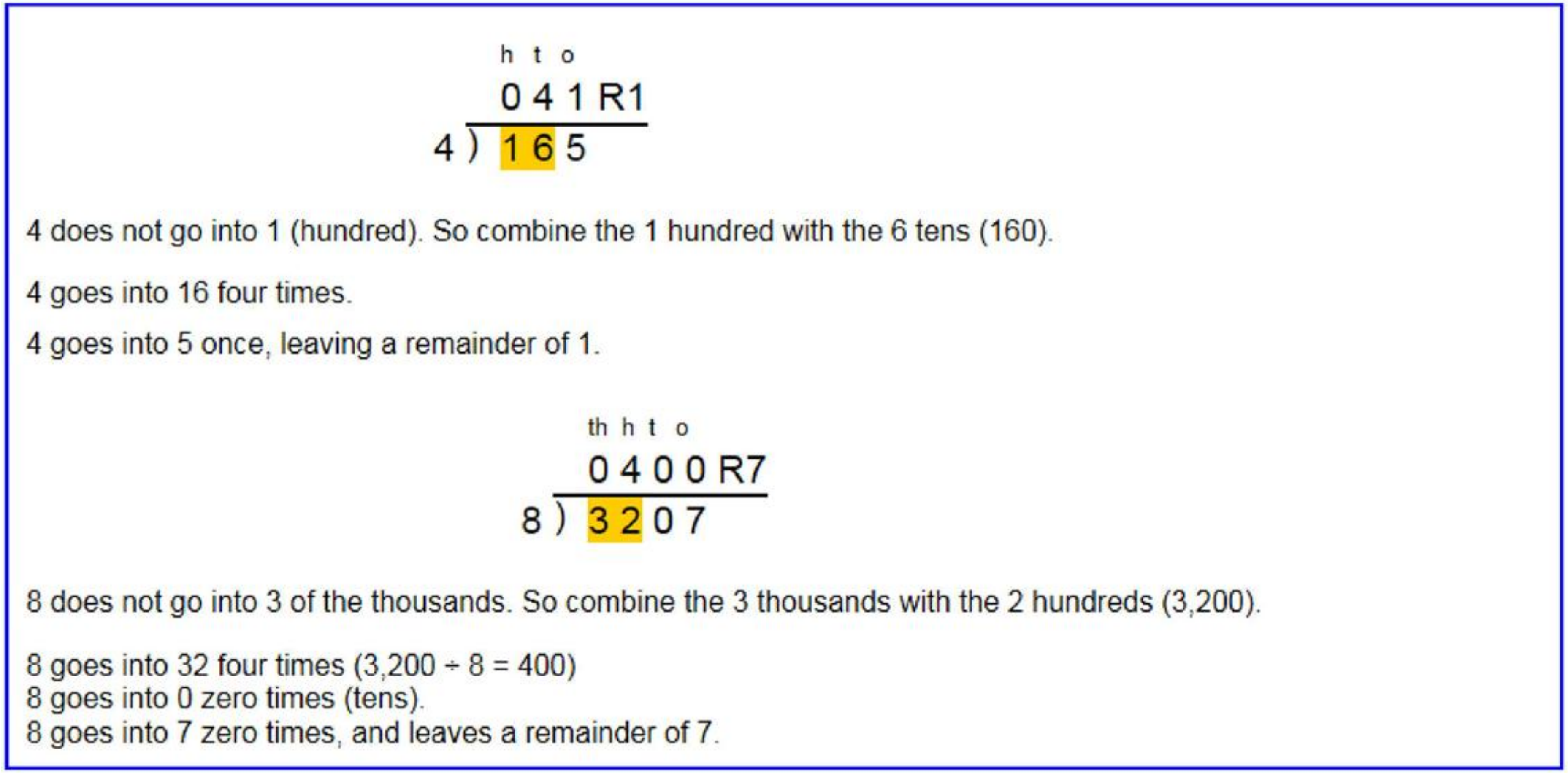
	 We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answeris 14.		

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, howcan you divide 615 by 5 without using short division? 	have £615 and share it equally between 5 bank accounts. How muchwill be in each account? 615 pupils need to be put into 5 groups. How many will be in eachgroup?	$5\overline{)615}$ 615 ÷ 5 = ☐ = 615 ÷ 5	What is the calculation?What is the answer? 

Long Division

Step 1—a remainder in the ones



h t o

0 4 1 R1

4) 1 6 5

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

th h t o

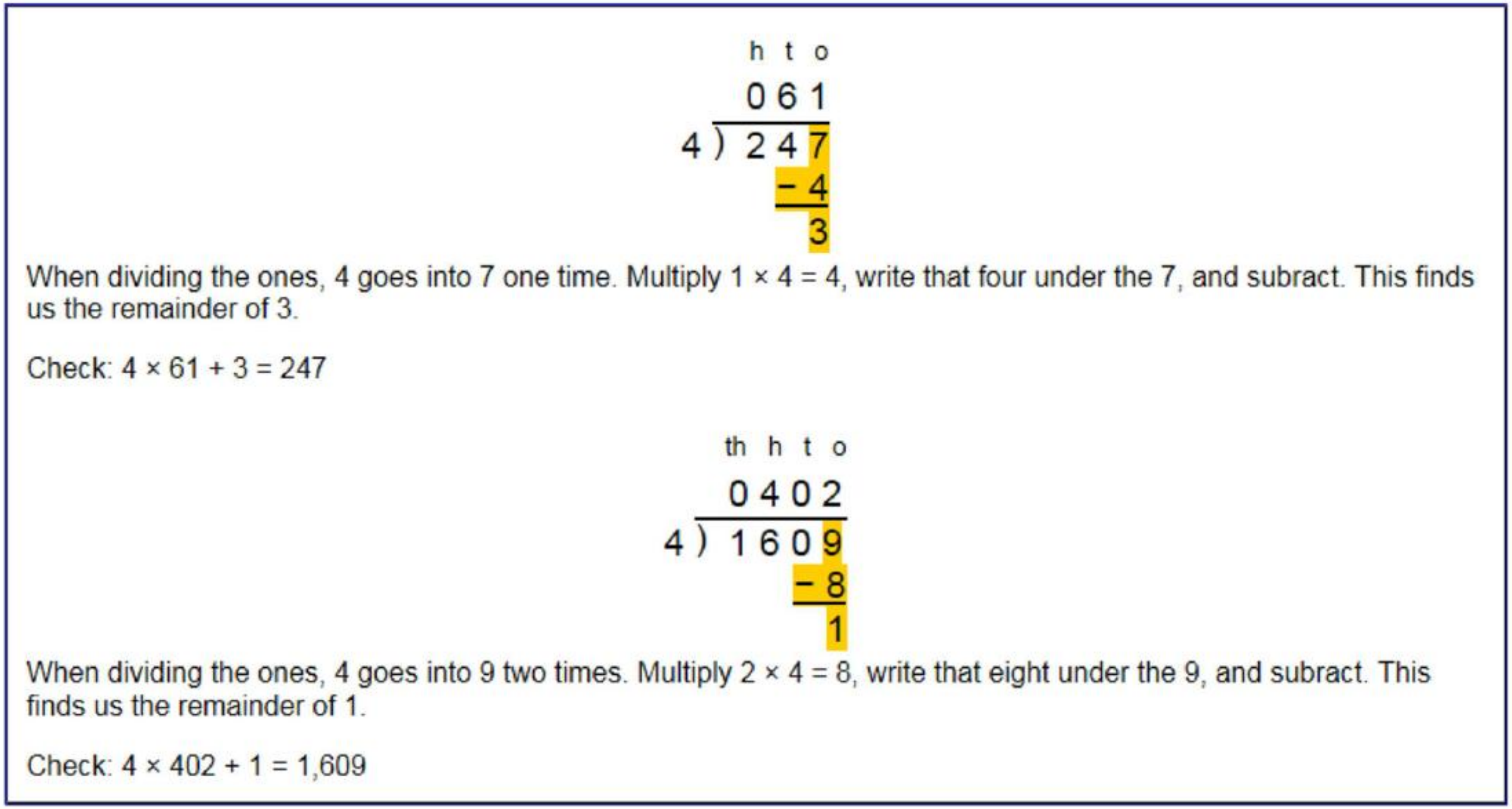
0 4 0 0 R7

8) 3 2 0 7

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Step 1 continued…..



h t o
0 6 1
4) 2 4 7
− 4
3

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

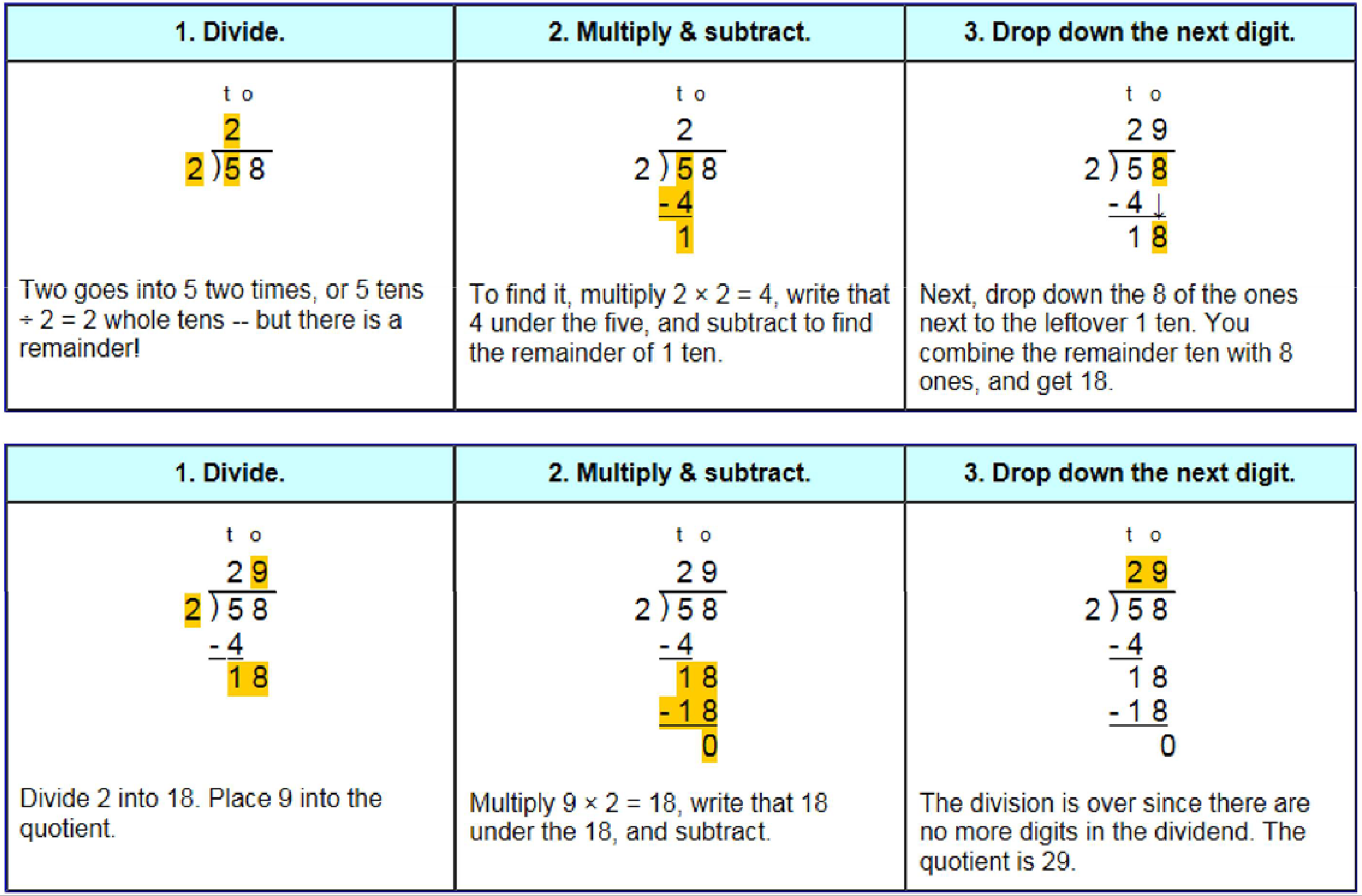
Check: 4 × 61 + 3 = 247

th h t o
0 4 0 2
4) 1 6 0 9
− 8
1

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

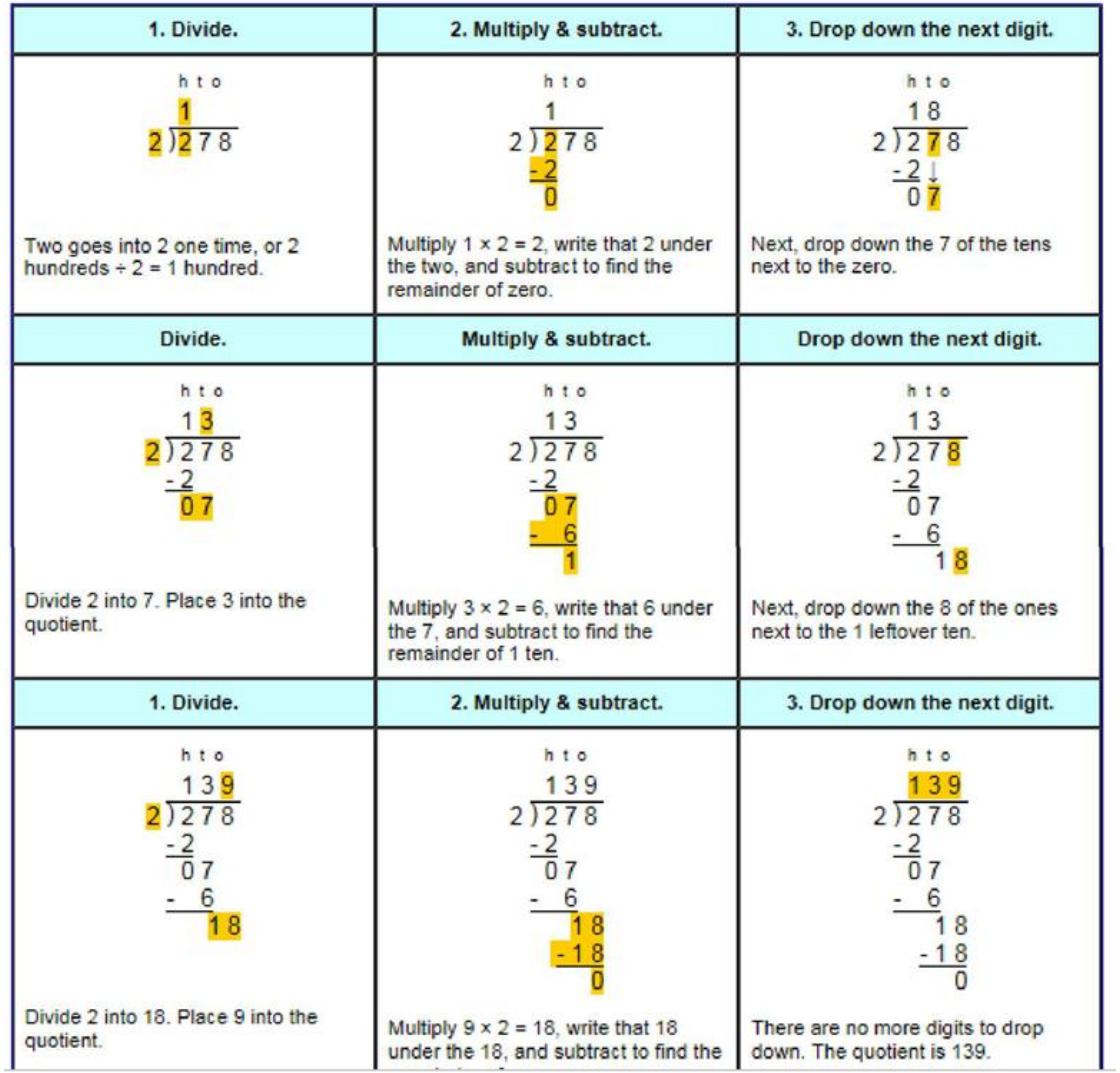
Step 2 – a remainder in the tens



1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{rcc} & t & o \\ & 2 & \\ 2\,\big) & \overline{5} & \overline{8} \end{array}$ Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	$\begin{array}{rcc} & t & o \\ & 2 & \\ 2\,\big) & \overline{5} & \overline{8} \\ & \underline{-4} & \\ & 1 & \end{array}$ To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	$\begin{array}{rcc} & t & o \\ & 2 & 9 \\ 2\,\big) & \overline{5} & \overline{8} \\ & \underline{-4} & \downarrow \\ & 1 & 8 \end{array}$ Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{rcc} & t & o \\ & 2 & 9 \\ 2\,\big) & \overline{5} & \overline{8} \\ & \underline{-4} & \\ & 1 & 8 \end{array}$ Divide 2 into 18. Place 9 into the quotient.	$\begin{array}{rcc} & t & o \\ & 2 & 9 \\ 2\,\big) & \overline{5} & \overline{8} \\ & \underline{-4} & \\ & 1 & 8 \\ & \underline{-1} & \underline{8} \\ & & 0 \end{array}$ Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	$\begin{array}{rcc} & t & o \\ & 2 & 9 \\ 2\,\big) & \overline{5} & \overline{8} \\ & \underline{-4} & \\ & 1 & 8 \\ & \underline{-1} & \underline{8} \\ & & 0 \end{array}$ The division is over since there are no more digits in the dividend. The quotient is 29.

Step 3 – a remainder in any of the place values

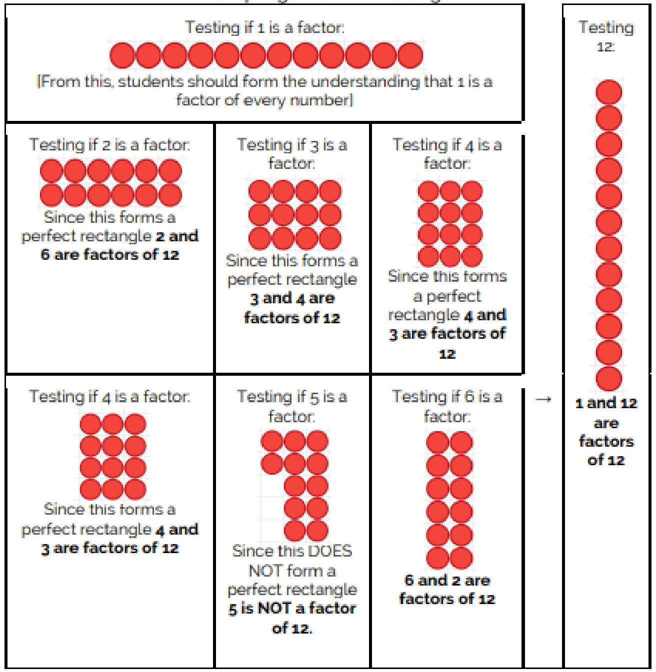
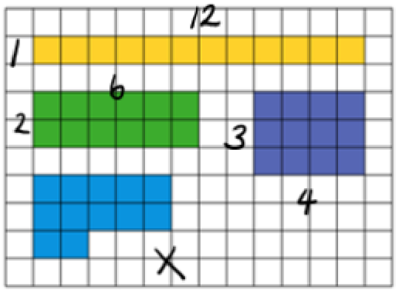
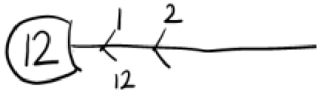
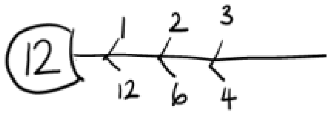


1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)278 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2)278 -2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 18 2)278 -2 ↓ 07 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 13 2)278 -2 07 Divide 2 into 7. Place 3 into the quotient.	h t o 13 2)278 -2 07 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 13 2)278 -2 07 - 6 18 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 139 2)278 -2 07 - 6 18 Divide 2 into 18. Place 9 into the quotient.	h t o 139 2)278 -2 07 - 6 18 -18 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the	h t o 139 2)278 -2 07 - 6 18 -18 0 There are no more digits to drop down. The quotient is 139.

Conceptual variation; different ways to ask pupils to solve 615 ÷ 5

KS2/KS3 Maths Topics – Number

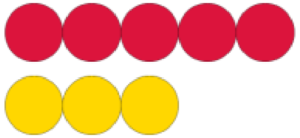
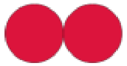
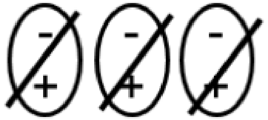
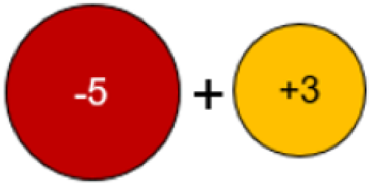
FACTORS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should use a manipulative (likely double sided counters or multi-link cubes) and maneuver them to understand factors and multiples	Students use the ideas formed in the physical stage to use the squares in exercise books to draw rectangles	Students should use a diagram to find the factors of the required number	Students should be able to list the factors immediately
Example(s)	**Example(s) - These should be seen in books**		
Find all of the factors of 12			
Students should represent the number using the manipulative  Students should then understand that amount of counters along and the amount of counters up are factors of the value. Students should then work **systematically**, adding one to the height and attempting to form a rectangle.  Testing if 1 is a factor: [From this, students should form the understanding that 1 is a factor of every number] Testing if 2 is a factor: Since this forms a perfect rectangle **2 and 6 are factors of 12** Testing if 3 is a factor: Since this forms a perfect rectangle **3 and 4 are factors of 12** Testing if 4 is a factor: Since this forms a perfect rectangle **4 and 3 are factors of 12** Testing if 4 is a factor: Since this forms a perfect rectangle **4 and 3 are factors of 12** Testing if 5 is a factor: Since this DOES NOT form a perfect rectangle **5 is NOT a factor of 12.** Testing if 6 is a factor: **6 and 2 are factors of 12** → Testing 12: **1 and 12 are factors of 12** Students should identify that the factors come in pairs and that they only need to be listed once. Factors of 12 are 1, 2, 3, 4, 6, 12	Students should use the squares in their books to sketch the value in rectangles. From the Physical stage, students should understand that the factors come in pairs and therefore only need to be drawn once.  Students should clearly explain why certain values are NOT factors. E.g. the light blue diagram above does not form a perfect rectangle, therefore 5 is NOT a factor of 12. Factors of 12 are 1, 2, 3, 4, 6, 12	Students should work systematically, starting at 1 and increasing by one each time and determining if it is a factor and the corresponding value.  Students should understand that as soon as they reach a number that is already in the diagram they have found all of the factors.  Students MUST then list the factors. Factors of 12 are 1, 2, 3, 4, 6, 12	Students will not need any form of diagram at this stage. Students should understand the meaning of factors and will likely be able to find factors in their head. Factors of 12 are 1, 2, 3, 4, 6, 12

DIRECTED NUMBER

GENERAL TEACHING & LEARNING POINTS	**The following apply to all four operations with negative numbers:** - Teachers should not use the term "minus", instead use the words "negative" and "subtract" - "Two negatives make a positive" should NEVER be used – this is not mathematically accurate - The idea of zero-pairs should be emphasised throughout as this will also be used in an algebra context - The idea that subtraction is the **additive inverse** should be shown, e.g. subtracting 3 is the same as adding -3 - Students should be confident with using double-sided counters to represent numbers prior to attempting four operations

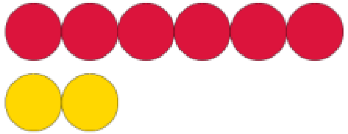
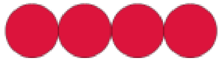
Addition

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to use **double-sided counters** to represent the two numbers and the idea of zero pairs.	Students should draw the manipulatives as a diagram in their books, clearly showing the zero-pairs.	Students should use the ideas in the Physical and Pictorial stages be able to determine, through using 'scaled' double-sided counter diagrams to determine if the answer is positive or negative.	Students should be able to state an answer without the use of a diagram.
Example(s)	**Example(s) - These should be seen in books**		
Calculate -5 + 3 "Negative 5 add 3"			
Students should use double-sided counters to represent the -5 using five -1 counters and then physically add three +1 counters.  Students should then identify the zero-pairs in their representation and remove them.  Students should then be able to identify their answer from their remaining counters. $-5 + 3 = -2$	Students should represent the calculation as in the Physical Stage, however, for short it may be beneficial for students to use – and + signs instead of drawing full circles. – – – – – + + + Students should then identify the zero-pairs in their representation **and circle them**. They could show they have been removed by crossing through them.  - - Students should then be able to identify their answer from their remaining symbols. $-5 + 3 = -2$	Students should represent this by drawing a larger negative circle plus a smaller positive circle. https://mathsbot.com/manipulatives/directedCounters may be useful for this representation. For weaker students a number line representation in addition may also be beneficial.  Students should be able to determine from the calculation that there are will be more negative counters than positive ones in this calculation, **therefore the answer must be negative.** $-5 + 3 = -2$	Students should be able to perform the calculation without any working. $-5 + 3 = -2$

Subtraction

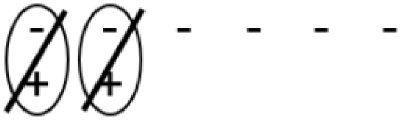
<table>
<tr><th>Stage 1
Physical</th><th>Stage 2
Pictorial</th><th>Stage 3
Semi-Abstract</th><th>Stage 4
Abstract</th></tr>
<tr><td>Students should be able to use double-sided counters to represent the two numbers and the idea of zero pairs.</td><td>Students should draw the manipulatives as a diagram in their books, clearly showing the zero-pairs.</td><td>Students should use the ideas in the Physical and Pictorial stages be able to determine, through using 'scaled' double-sided counter diagrams to determine if the answer is positive or negative.</td><td>Students should be able to state an answer without the use of a diagram.</td></tr>
<tr><td>Example(s)</td><td colspan="3">Example(s) - These should be seen in books</td></tr>
<tr><td colspan="4">Calculate -6 – -2
“Negative 6 subtract -2”</td></tr>
<tr><td>Students should use double-sided counters to represent the -6.
https://mathsbot.com/manipulatives/doubleSidedCounters may be useful for this representation.

Students should then understand that subtracting -2 is equivalent to +2.

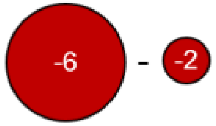
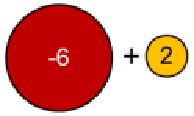
Students should then add these counters to their representation.

Students should then identify the zero-pairs in their representation and remove them.

Students should then be able to identify their answer from their remaining counters.

-6 - -2 = -4</td><td>Students should represent the calculation as in the Physical Stage, however, for short it may be beneficial for students to use – and + signs instead of drawing full circles.

– – – – – –
+ +

Students should then identify the zero-pairs in their representation <u>and circle them</u>. They could show they have been removed by crossing through them

Students should then be able to identify their answer from their remaining symbols.

-6 - -2 = -4</td><td>Students should represent this by drawing a larger negative circle subtract a smaller negative circle.
https://mathsbot.com/manipulatives/directedCounters may be useful for this representation.

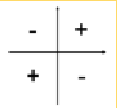
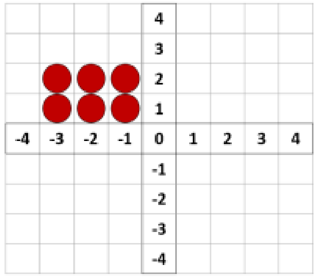
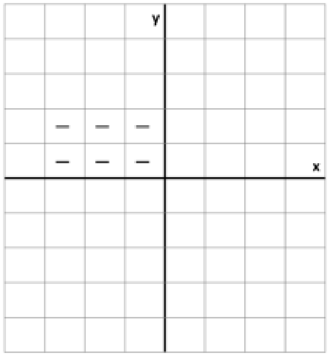
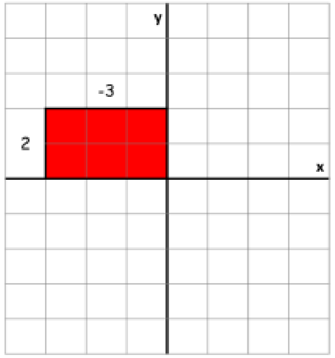
For weaker students a number line representation in addition may also be beneficial.

Students should then apply the additive inverse as they have done in previous stages and re-draw.

Notice the size of the second circle does not change.

Students should be able to determine from the calculation that there are will be more negative counters than positive ones in this calculation, <u>therefore the answer must be negative.</u>

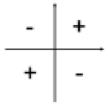
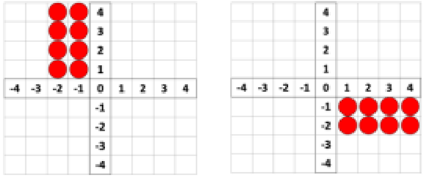
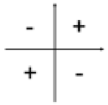
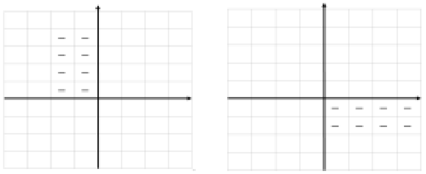
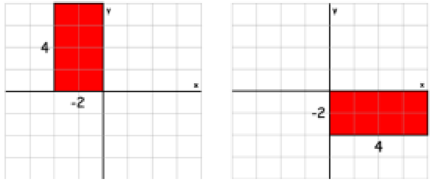
-6 - -2 = -4</td><td>Students should be able to perform the calculation without any working.

-6 - -2 = -4</td></tr>
</table>

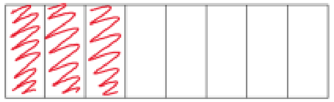
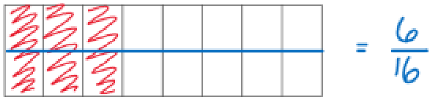
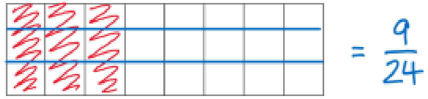
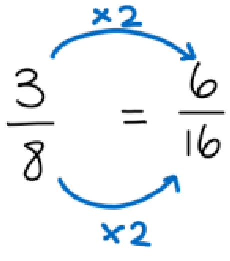
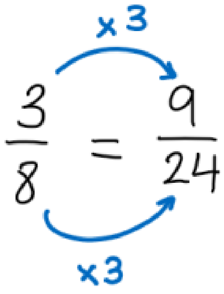
Multiplication

GENERAL TEACHING & LEARNING POINTS	- Students should be confident using factors and multiples and the use of a co-ordinate grid. - Students should understand that the opposite side of the x or y axis involves multiplying by -1 (flipping the counters over) 		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to use **double-sided counters** and a numbered grid to represent the problem as an area model.	Students should draw the manipulatives as a diagram in their books.	Students should use the ideas in the Physical and Pictorial stages be able to determine an answer, through using the area of a rectangle model.	Students should be able to state an answer without the use of a diagram.
Example(s)	**Example(s) - These should be seen in books**		
Calculate -3 x 2 "Negative 3 multiplied by 2"			
For consistency students could represent the first number on the x-axis and the second the y-axis. Students should use the grid as shown below:  The counters are -3 along the x axis and +2 lots of this (so +2 on the y-axis). Students should then be able to identify their answer from the counters on their grid -3 x 2 = -6	Students should draw a small grid in their exercise books and place either + or – symbols in each square to represent the same idea as in the physical stage.  Students should then be able to identify their answer from the signs in their grid -3 x 2 = -6	Students should draw a similar diagram to those in the Physical and Pictorial stage, however at this stage students should represent the calculation through an area model.  Students should then apply their knowledge of understanding -3 x 2 = -6	Students should be able to perform the calculation without any working. -3 x 2 = -6

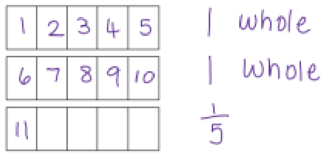
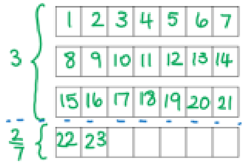
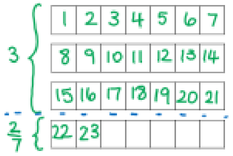
Division

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to use **double-sided counters** and a numbered grid to represent the problem as an area model.	Students should draw the manipulatives as a diagram in their books.	Students should use the ideas in the Physical and Pictorial stages be able to determine an answer, through using the area of a rectangle model.	Students should be able to state an answer without the use of a diagram.
Example(s)	**Example(s) - These should be seen in books**		
Calculate -8 ÷ -2 "Negative 8 divided by negative 2"			
Students will be confident with the + - grid below.  Students should know that for this calculation -8 represents the counters required, and this would be represented in quadrant 2 or 4. Student should understand that one dimension needs to be -2. Either representation below could be used.  Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram. -8 ÷ -2 = 4	Students will be confident with the + - grid below.  Students should know that for this calculation -8 represents the counters required, and this would be represented in quadrant 2 or 4. Student should understand that one dimension needs to be -2. Either representation below could be used.  Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram. -8 ÷ -2 = 4	Students should represent the calculation as an area as with Dividing Numbers. Either representation below could be used.  Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram. -8 ÷ -2 = 4	At this stage students should be able to perform the calculation without the need for any working. -8 ÷ -2 = 4

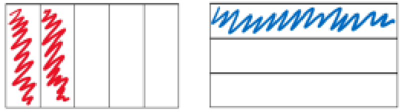
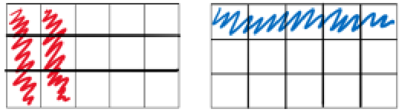
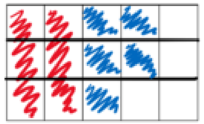
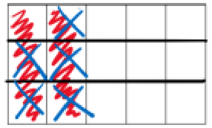
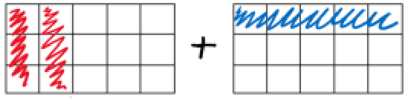
EQUIVALENT FRACTIONS

<table>
<tr><th>Stage 1
Physical</th><th>Stage 2
Pictorial</th><th>Stage 3
Semi-Abstract</th><th>Stage 4
Abstract</th></tr>
<tr><td>N/A</td><td>Students should then be able to represent each fraction as a diagram, one split horizontally and the other vertically</td><td>Students should use a scale factor approach to find equivalent fractions.</td><td>Students should be able to state equivalent fractions.</td></tr>
<tr><td>Example(s)</td><td colspan="3">Example(s) - These should be seen in books</td></tr>
<tr><td colspan="4">Write down three fractions that are equivalent to $\frac{3}{8}$</td></tr>
<tr><td>This may be added at a later date.</td><td>Students should represent the $\frac{3}{8}$ fraction using a diagram, split either horizontally OR vertically.

Students should then understand that as long as this is equally split horizontally (or vertically if the original diagram is split horizontally) then the fractions will be equivalent to $\frac{3}{8}$

 </td><td>Students should confidently understand that if the numerators and denominators are multiplied or divided by the same factor then the two fractions will be equivalent.
This multiplication should be shown at this stage.

 </td><td>At this stage students should be able to immediately state fractions that are equivalent to $\frac{3}{8}$</td></tr>
</table>

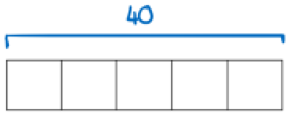
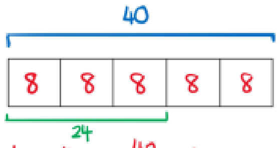
CONVERTING BETWEEN MIXED NUMBERS AND IMPROPER FRACTIONS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should represent the initial quantity using a bar diagram.	Students at this stage should be able to determine the number of parts in the wholes plus any extras.	Students should be able to state the equivalent improper fraction/mixed number without any working.
Example(s)	**Example(s) - These should be seen in books**		
Write $\frac{11}{5}$ as a mixed number.			
This may be added at a later date.	Students should understand that, since the denominator is 5, each whole one is split into 5 pieces. Students should draw a diagram to show the 11 parts.  $= 2\frac{1}{5}$ Students should then identify the whole parts and the fraction parts and combine.	Students should be encouraged to think of how many **whole** 5s are in 11, and what the remainder is. $\frac{11}{5} = 2$ remainder 1 $= 2\frac{1}{5}$	At this stage students should be able to immediately state the answer. $\frac{11}{5} = 2\frac{1}{5}$
Write $3\frac{2}{7}$ as a mixed number.			
This may be added at a later date.	Students should understand that, since the denominator is 7, each whole one is split into 7 pieces. Students should then count the number of parts.  $\frac{23}{7}$	Students should **partly** rely on the diagram at this stage. Students should be able to determine the number of sevenths in the three whole ones and add the extras.  $3 \times 7 = 21$ $21 + 2 = 23$ $= \frac{23}{7}$	$3\frac{2}{7} = \frac{23}{7}$

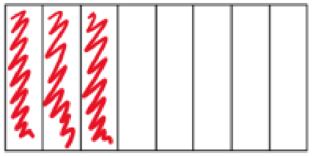
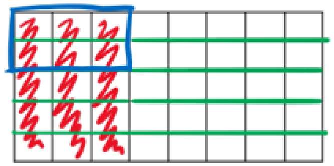
ADDING AND SUBTRACTING FRACTIONS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should then be able to represent each fraction as a diagram, one split horizontally and the other vertically	Students should be able to determine the LCM of the two denominators.	Students should be able to add and subtract fractions without the need for a diagram; understanding the need for the LCM.
Example(s)	**Example(s) - These should be seen in books**		
Calculate $\frac{2}{5}+\frac{1}{3}$			
This may be added at a later date.	Students should represent each of the two fractions as separate diagrams. ONE DIAGRAM SHOULD BE SPLIT VERTICALLY AND THE OTHER HORIZONTALLY.  Students should then identify the LCM in order to find a common denominator and split each representation into this number of parts - See Equivalent Fractions.  Students should then understand that now all the small parts in the diagram are the same size, these can be combined. For addition, students should add the parts in both diagrams to a single diagram.  $= \frac{11}{15}$ [For subtraction, students should cross through a part in the first diagram for each part in the second diagram] $\frac{2}{5} - \frac{1}{3}$ 	Students should be able to determine the LCM of the two denominators, 15 in this case. Students should then draw diagrams to represent these.  $\frac{2}{5} \overset{\times 3}{=} \frac{6}{15}$ $\frac{1}{3} \overset{\times 5}{=} \frac{5}{15}$ $\frac{6}{15} + \frac{5}{15}$ $= \frac{11}{15}$	$\frac{2}{5} + \frac{1}{3}$ $\times 3$ $\times 3$ $\times 5$ $\times 5$ $\frac{6}{15} + \frac{5}{15}$ $= \frac{6+5}{15}$ $= \frac{11}{15}$

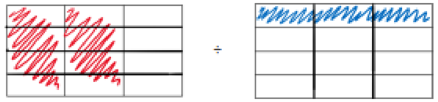
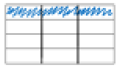
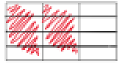
FRACTIONS OF AMOUNTS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should use a proportion idea first identifying a unit fraction and then scaling up.	Students should use a simple written method that shows finding a unit fraction and then multiplying.
Example(s)	**Example(s) - These should be seen in books**		
Find $\frac{3}{5}$ of 40			
This may be added at a later date.	Students should represent the problem as a bar model, splitting the whole bar into the number of parts shown in the denominator and clearly showing the total.  Students should then work out the value of one part, students should then add this to their diagram.  $\frac{1}{5}$ of $40 = \frac{40}{5} = 8$ Students should then clearly show how they have obtained the $\frac{3}{5}$  $\frac{1}{5}$ of $40 = \frac{40}{5} = 8$ $\frac{3}{5}$ of $40 = 8 \times 3 = 24$	Students should find $\frac{1}{5}$ and then multiply this by 3. $$\begin{tikzcd} \frac{1}{5} \text{ of } 40 = \frac{40}{5} \arrow[d, bend right, "\times 3"'] & =8 \arrow[d, bend left, "\times 3"] \\ \frac{3}{5} \text{ of } 40 = & 24 \end{tikzcd}$$	Students should show find $\frac{1}{5}$ and then multiply this by 3 as a single calculation. $\frac{40}{5} \times 3 = 24$

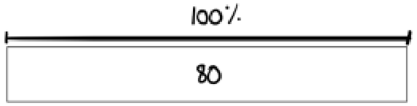
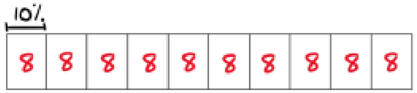
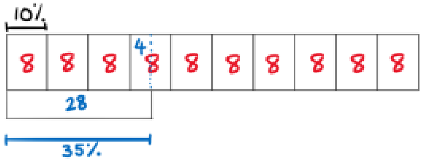
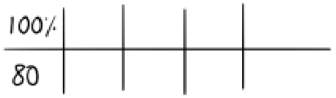
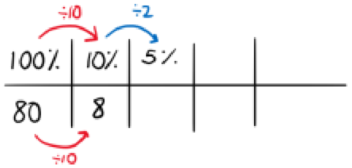
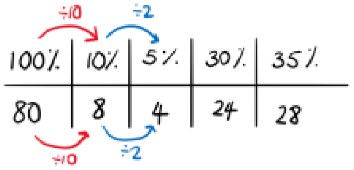
MULTIPLYING FRACTIONS

GENERAL TEACHING & LEARNING POINTS	- To be consistent with diagrams, students should understand that multiplying fractions means the second a fraction OF the first, e.g. $\frac{1}{6} \times \frac{1}{2}$ means $\frac{1}{2}$ of $\frac{1}{6}$		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should clearly show their written method, simplifying their answer.	Students at this stage should look for common factors to cancel to avoid larger numerators and denominators.
Example(s)	**Example(s) - These should be seen in books**		
Calculate $\frac{3}{8} \times \frac{2}{5}$			
This may be added at a later date.	Students should draw a diagram to represent the $\frac{3}{8}$  Students should then find $\frac{2}{5}$ of the shaded part. Students should divide this into 5 part.  Students should circle the $\frac{2}{5}$ $= \frac{6}{40}$ $= \frac{3}{20}$	Students should complete the calculation without the need for a diagram. It is important that students include the $\frac{3\times2}{8\times5}$ step to clearly show their understanding. $\frac{3}{8} \times \frac{2}{5}$ $= \frac{3 \times 2}{8 \times 5}$ $= \frac{6}{40}$ $= \frac{3}{20}$	Students should be encouraged to spot possible calculations after the first step, as shown below. $\frac{3}{8} \times \frac{2}{5}$ $= \frac{3 \times \cancel{2}^{1}}{{}_{4}\cancel{8} \times 5}$ $= \frac{3 \times 1}{4 \times 5}$ $= \frac{3}{20}$

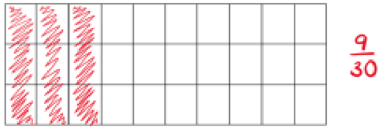
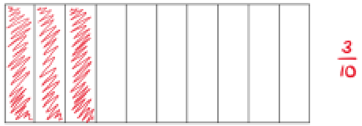
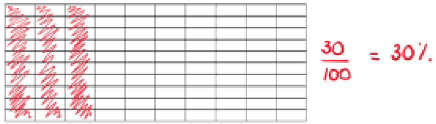
DIVIDING FRACTIONS

GENERAL TEACHING & LEARNING POINTS	- "Keep Flip Change" should never be used - Students should be encouraged to "multiply by the reciprocal" - Students should be encouraged to think of $\frac{2}{3} \div \frac{1}{4}$ as "How many $\frac{1}{4}$ are there in $\frac{2}{3}$"		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should clearly show their written method, simplifying their answer.	Students at this stage should clearly show division by a fraction is equivalent to multiplying by its reciprocal.
Example(s)	**Example(s) - These should be seen in books**		
Calculate $\frac{2}{3} \div \frac{1}{4}$			
This may be added at a later date.	Students should represent each fraction in the calculation as separate diagrams, splitting one vertically and the other horizontally.  Students should use their knowledge of Equivalent Fractions to split their diagrams so that the parts in each diagram are equal sizes.  Students should understand that students need to determine how many  in  Students should then circle the groups of three parts  $= 2\frac{2}{3}$	Students should understand that if they multiply both fractions by the reciprocal of the second, this reduces the calculation to a multiplication, and division by 1. $\frac{2}{3} \div \frac{1}{4}$ $\times\frac{4}{1}$ $\quad$ $\times\frac{4}{1}$ $\left(\frac{2}{3}\times\frac{4}{1}\right) \div \left(\frac{1}{4}\times\frac{4}{1}\right)$ $= \frac{8}{3} \div 1$ $= \frac{8}{3}$ $= 2\frac{2}{3}$	Students should represent this multiplication by the reciprocal as a single calculation. $\frac{2}{3} \div \frac{1}{4}$ $= \frac{2}{3} \times \frac{4}{1}$ $= \frac{8}{3}$ $= 2\frac{2}{3}$

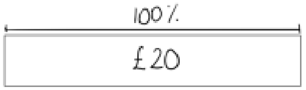
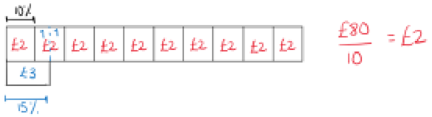
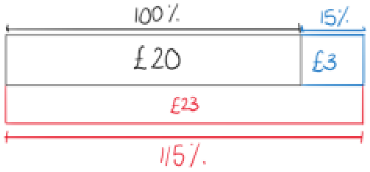
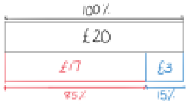
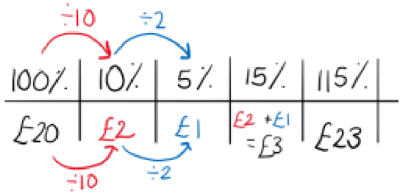
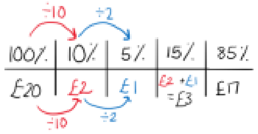
PERCENTAGE OF AMOUNTS

GENERAL TEACHING & LEARNING POINTS	•		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should use a written method, **set out using a proportion table**.	Students should use a written, **multiplier**, method in order to calculate the required percentage.
Example(s)	**Example(s) - These should be seen in books**		
Calculate 35% of 80			
This may be added at a later date.	Students should use a bar model to represent the initial 100%.  Students should then split their bar model into a suitable number of parts.  $\frac{80}{10} = 8$ Students should then determine the correct percentage by using a second bar underneath  $\frac{80}{10} = 8$	Students should represent the initial amount at 100% in a proportion table.  Students should then calculate a suitable percentage(s) to use to calculate the final amount, (10% and 5% in this case).  Students should complete their proportion table to obtain the required final percentage. Students should clearly state their final answer.  35% of 80 = 28	Students should understand that 'of' means multiply and determine the multiplier. $0.35 \times 80 = 28$

ONE QUANTITY AS A PERCENTAGE OF ANOTHER

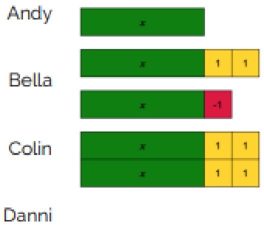
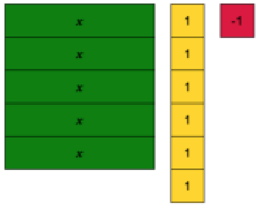
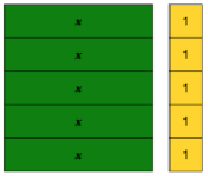
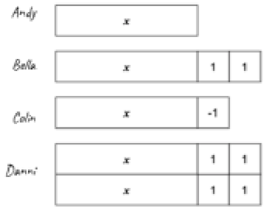
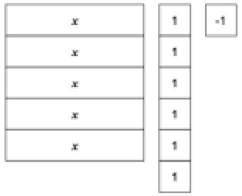
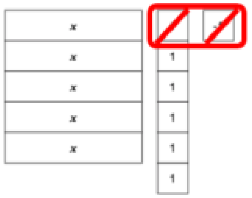
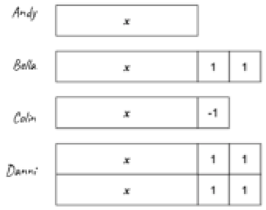
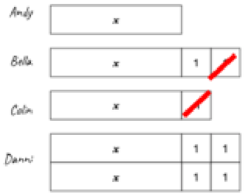
GENERAL TEACHING & LEARNING POINTS	•		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should use a written method using equivalent fractions.	Students should understand that they can write their calculation as a single line of working, which could be entered into a calculator if permitted.
Example(s)	**Example(s) - These should be seen in books**		
Zuzanna scored 9 out of 30 on a maths test. Work out her score as a percentage.			
This may be added at a later date.	Students should represent the problem as a diagram. They should divide the total amount into an appropriate amount, usually a factor of 100.  Students should understand that percentage means 'per 100'. They should then split their diagram into 100 parts. It may be appropriate to split their diagram using two stages.  Followed by 	Students should use their understanding obtained in the Pictorial Stage and the idea that this relates back to equivalent fractions, to show their working as below: $$\begin{tikzcd} \frac{9}{30} \arrow[rr, bend left, "\div 3"] \arrow[rr, bend right, "\div 3"'] & = & \frac{3}{10} \arrow[rr, bend left, "\times 10"] \arrow[rr, bend right, "\times 10"'] & = & \frac{30}{100} & = 30\% \end{tikzcd}$$	$\frac{9}{30} \times 100 = 30\%$

PERCENTAGE INCREASE/DECREASE

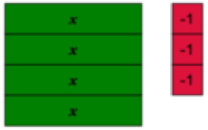
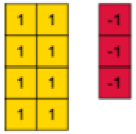
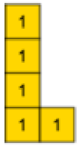
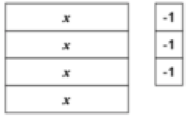
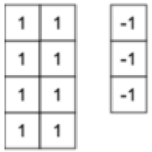
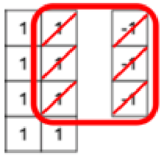
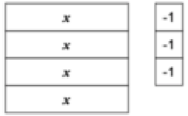
GENERAL TEACHING & LEARNING POINTS	• Students should understand that increasing (or decreasing) by a percentage is equivalent to finding a percentage of an amount above (or below) 100%.		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should use a proportion table to find the required percentage.	Students should be able to use a calculator, through a multiplier, to determine the final amount.
Example(s)	**Example(s) - These should be seen in books**		
Increase £20 by 15%			
This may be added at a later date.	Students should use a bar model to represent the initial 100%.  Students should then use the method shown in Finding Percentage of an Amount, to find the percentage to increase (or decrease) by.  Students should then add on the £3 to the end of their diagram, clearly labelling this as 115%.  *Decrease £20 by 15%* 	Students should use their understanding of proportionality to set their working out in a proportion table, as shown below.  *Decrease £20 by 15%* 	Students should understand that 'of' means multiply and determine the multiplier. $1.15 \times 20 = £23$ *Decrease £20 by 15%* $0.85 \times 20 = £17$

Algebra

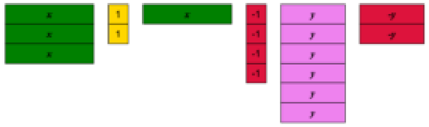
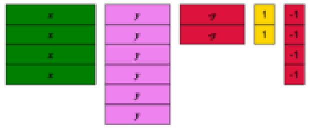
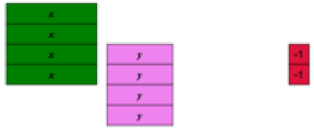
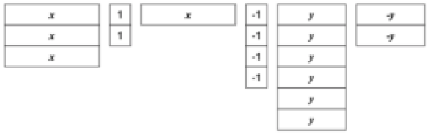
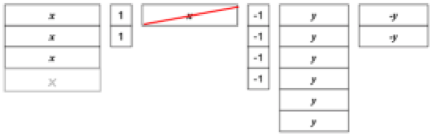
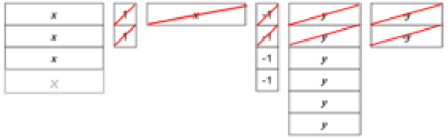
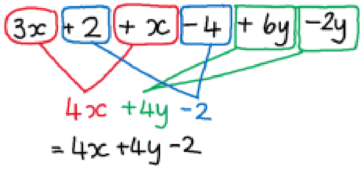
FORMING EXPRESSIONS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulative (cuisenaire rods or algebra tiles).	Students should draw the manipulatives as a diagram in their books, clearly collecting like terms and identifying the zero-pairs.	Students should use a diagram and identify the zero-pairs and cross them out.	Students should use algebra only to be able to collect like terms to find the total.
Example(s)	**Example(s) - These should be seen in books**		
Andy is years' old. Bella is two years' older than Andy. Colin is one year younger than Andy. Danni is twice as old as Bella. Write an expression for the **total** age of all four people.			
Represent each person's age using cuisenaire rods, place each person above each other to show comparisons.  Students should collect like terms (see Collecting Like Terms section)  Students should then use the idea of **zero-pairs** to simplify  Students should understand that this represents	Represent each person's age using cuisenaire rods, place each person above each other to show comparisons.  Students should collect like terms (see Collecting Like Terms section)  Students should then use the idea of **zero-pairs** to simplify. Students should circle, label and cross out the zero-pairs  Students should then be able to count the 's and the 1's and that this represents	Represent each person's age using cuisenaire rods, place each person above each other to show comparisons.  Students should then use the idea of **zero-pairs** to simplify, without the need to show the like terms together. Students should circle, label and cross out the zero-pairs.  Students should then be able to count the remaining 's and 1's and that this represents	Students should represent each person's age as an algebraic expression. **A** x **B** x+2 **C** x-1 **D** 2x+4 *This could be represented as 2(x+2) depending on student's ability* Students should then represent this as one expression clearly showing each expression either added or subtracted. $x + (x+2) + (x-1) + (2x+4)$ Notice that each expression is in brackets to ensure that students do not make sign errors with negatives.

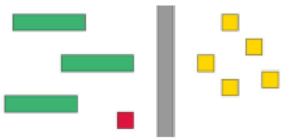
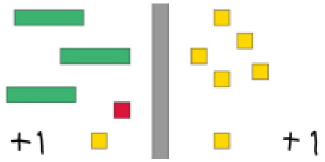
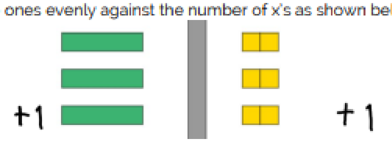
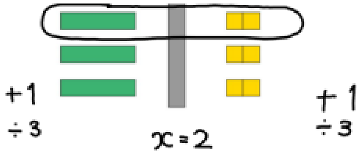
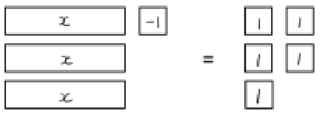
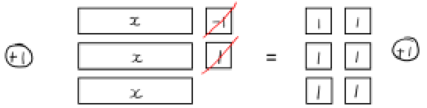
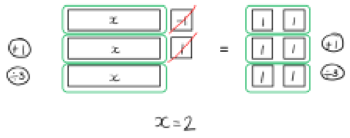
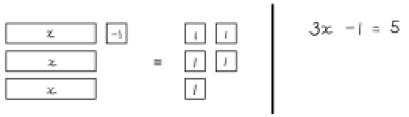
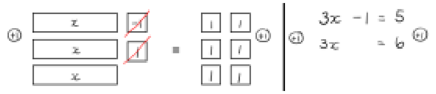
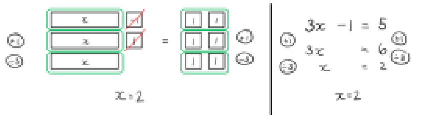
SUBSTITUTION INTO ALGEBRAIC EXPRESSIONS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to algebra tiles to represent the expression in the problem, then replace the variable tiles with numerical tiles.	Students should draw the manipulatives as a diagram in their books, clearly replacing the variable with 1 or -1 tiles.	Students now be able to use the initial diagram and replace the variables with a value.	Students should now be able to apply their understanding of algebraic notation to substitution. At this stage, students will not need a diagram.
Example(s)	**Example(s) - These should be seen in books**		
If $x = 3$ find the value of the expression $4x - 3$			
Using Algebra Tiles to represent the expression  Students should then replace x tiles with 3 1 tiles.  Students should then use the idea of **zero-pairs** to simplify  Students can then count the tiles to determine the answer. **5**	Students should draw the algebra tiles representation in their books $4x - 3 =$  Students should clearly state (through a diagram) the value of each variable:  Students should then redraw their diagrams replacing the x bars with 3 1 tiles. Where possible, students should be encouraged to make the number of 1 tiles the same size as the x tile it is replacing.  Students should then use the idea of **zero-pairs** to simplify. Students should circle, label and cross out the zero-pairs.  Students can then count the tiles to determine the answer. **5**	Students should draw the algebra tiles representation in their books $4x - 3 =$  Students should clearly state (through a diagram) the value of each variable, using a number rather than 1s and -1s:  $= 3$ Students should then replace the x tiles by crossing out the x in their diagram and replacing it with the value of x. $4x - 3 =$  Students should then calculate the value of the expression: $2 + 2 + 2 + 2 - 3$ $= 4 \times 2 - 3$ = **5**	Students should be able to re-write the algebraic expression with clear algebraic understanding $4x - 3$ $= 4 \times x - 3$ $= 4 \times 2 - 3$ $= 8 - 3$ = **5** Higher attaining students should be discouraged from using the multiplication sign as above and instead should be encouraged to use brackets: $4x - 3$ $= 4(2) - 3$ $= 8 - 3$ = **5**

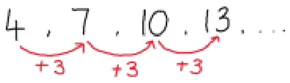
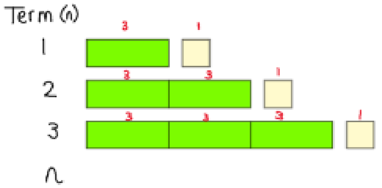
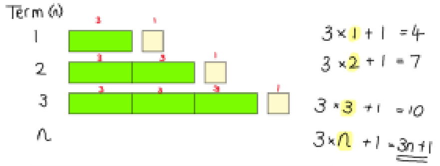
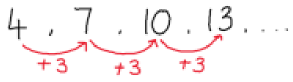
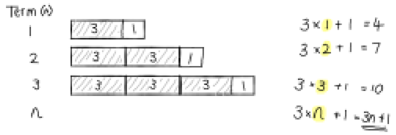
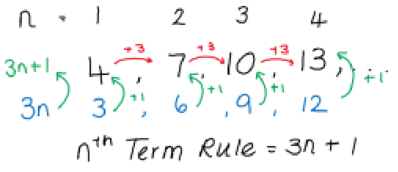
COLLECTING LIKE TERMS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to algebra tiles to represent the expression in the problem, then collect like tiles and apply the idea of zero-pairs	Students should draw the manipulatives as a diagram in their books, clearly collecting like terms and identifying the zero-pairs.	Students now be able to use the initial diagram and replace the variables with a value.	Students should now be able to apply their understanding of like terms without the need for any diagram.
Example(s)	**Example(s) - These should be seen in books**		
Simplify $3x + 2 + x - 4 + 6y - 2y$			
Using Algebra Tiles to represent the expression  Students should then physically move the tiles so that all identical tiles are together.  Students should use the idea of zero-pairs to simplify the expression.  Students should then identify the simplification from the representation. **$4x + 4y - 2$**	Students should draw the Algebra Tiles in their books, clearly labelling each tile.  Students should then collect like terms cross out a term and adding it to the remaining tiles of the same like term.  Students should then identify and cross through terms that make zero-pairs.  Students should then be able to identify the simplification from this final representation. **$4x + 4y - 2$**	Students should first identify, circle and join like terms. By this stage students should understand the term includes the sign in front of the algebra part.  Students should be able to clearly justify why the terms $4x$, -2 and $+4y$ do not simplify.	Students should be able to collect like terms without the need to circle the like terms. $3x + 2 + x - 4 + 6y - 2y$ $= 4x + 4y - 2$

SOLVING LINEAR EQUATIONS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulative algebra tiles.	Students should draw the manipulatives as a diagram in their books.	Students should be able to relate the diagrams of the algebra tiles to the equations.	Students should be able to use a formal balancing method to solve the equation.
Example(s)	**Example(s) - These should be seen in books**		
Solve the equation $3x - 1 = 5$			
Students should use Algebra Tiles to represent the equation. Students should lay tiles on a **mini-whiteboard** so that students can write down their process. Students should then use the idea of zero-pairs to eliminate ones on one side.  Students should note down on each side of the equation what process they have done to keep the equation balanced.  Students should then remove any zero-pairs and arrange the ones evenly against the number of x's as shown below  Students should then determine how to find the value of a single x, write this process below any previous operations, and circle this 	Students should draw the algebra tiles in their books. Students should draw the x tiles first and then work in columns to add the ones, as shown below:  Students could use highlighters to make drawing the tiles easier and quicker. Students should then use zero-pairs in order to eliminate the 1 or -1 tiles from one side of the equation. Students should note down what they have added to both sides of the equation in order to do this. Students should then cross out any zero-pairs.  If students have lined the x and 1s in columns systematically then they will be able to easily identify the value of 1 x tile. Students should clearly write down that they have divided/multiplied below the last operation. Some students may find it useful to circle the groups which give the value of one x. 	Students should represent the equation in the same way as in the pictorial stage, but with the addition of a formal balancing method also shown.  Students should then form zero-pairs as in the pictorial stage. Students should also include what they have done in the balancing method.  Students should then use a similar diagram to the one used in the pictorial stage.  The student's final solution should be clearly written underneath any diagram or working.	At this stage students should be comfortable with solving equations without the need for a diagram and can solely rely on a formal balancing method. Students however, at least initially, should show the operation that they have used in each stage of working. $3x - 1 = 5$ (+1) (+1) $3x = 6$ (÷3) (÷3) $x = 2$ $x = 2$

LINEAR SEQUENCES

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulative, e.g. Cuisenaire Rods.	Students should draw the manipulatives as a diagram in their books.	Students should be able to identify the term-to-term rule and comparing this to the relevant times table.	Students should be able to determine the nth term rule by mentally comparing the sequence to the relevant times table.
Example(s)	**Example(s) - These should be seen in books**		
Work out the nth term rule for the sequence 4, 7, 10, 13, ...			
It will be beneficial if students use the manipulatives on top of a whiteboard for this stage. Students should identify that the sequence increases by 3 each time.  Students should then know that the sequence is linked to the 3 times table. Students should then represent this using the manipulative. Since the sequence increases by 3, the 3 block will be needed.  Notice the term is also included and the value of each block is written above.  Students should then write each term as a multiple of the term-to-term rule, as shown on the right above.	Students should identify that the sequence increases by 3 each time.  Students should then know that the sequence is linked to the 3 times table.  It may be useful for weaker students to shade in the term-to-term blocks to make them easier to count and distinguish.	 Students should first write in the term numbers above the sequence. Students should then determine the term-to-term rule, shown in **red** above. Students should then know that the sequence is linked to the 3 times table and therefore the sequence 3n is needed. This should be written below, shown in **blue** above. Students should then determine how they obtain the sequence from the 3n row, this is shown in **green** above. This can be done either as shown above or by subtracting (this may be useful for quadratics).	Students may be able to determine the nth term rule immediately at this stage and state the answer of 3n+1. Students however MUST be able to clearly justify why by explaining that is the sequence is "one more than the three times table, therefore the nth term rule is 3n+1."

LAWS OF INDICES

<table>
<tr><td>GENERAL TEACHING & LEARNING POINTS</td><td>The following apply to the multiplication, division and power on power rule:
• Teachers should ensure thorough understanding through writing out the meaning of each term as repeated multiplication at first instance.
• When confident through using this repeated multiplication, only then should students be shown the rule</td></tr>
</table>

Multiplication Law

<table>
<tr><td>Stage 1
Physical</td><td>Stage 2
Pictorial</td><td colspan="2">Stage 3
Semi-Abstract</td><td>Stage 4
Abstract</td></tr>
<tr><td>N/A</td><td>N/A</td><td colspan="2">Students should understand the idea of an index and write out the meaning of each term.</td><td>Students should now be able to apply their understanding of like terms without the need to write out any terms.</td></tr>
<tr><td colspan="2">Example(s)</td><td colspan="3">Example(s) - These should be seen in books</td></tr>
<tr><td colspan="5">Write 4a x 3a² as a single power of a.</td></tr>
<tr><td colspan="2">NOT APPLICABLE</td><td>It may be useful to use colour to differentiate between the terms in the question.
Students at this stage should clearly write out what each term means as a repeated multiplication.
$\underline{4a} \times \underline{3a^2}$
$= 4 \times a \times 3 \times a \times a$
Students should then understand that multiplication is associative and therefore the order of multiplication is irrelevant.
$\underline{4a} \times \underline{3a^2}$
$= 4 \times a \times 3 \times a \times a$
$= 4 \times 3 \times a \times a \times a$
Students should then evaluate
$\underline{4a} \times \underline{3a^2}$
$= 4 \times a \times 3 \times a \times a$
$= 4 \times 3 \times a \times a \times a$
$= 12 \times a^3$
Then leading to a final answer, without a multiplication sign
$\underline{4a} \times \underline{3a^2}$
$= 4 \times a \times 3 \times a \times a$
$= 4 \times 3 \times a \times a \times a$
$= 12 \times a^3$
$= 12a^3$</td><td>Only when confident with the previous layout, students should be able to write their working as:
$4a \times 3a^2$
$= 4 \times 3 \times a^{1+2}$
$= 12a^3$</td><td>Students should be able to state the final answer to the question without the need for any working.
$4a \times 3a^2$
$= 12a^3$</td></tr>
</table>

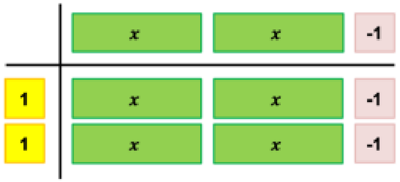
Division Law

<table>
<tr><th>Stage 1
Physical</th><th>Stage 2
Pictorial</th><th colspan="2">Stage 3
Semi-Abstract</th><th>Stage 4
Abstract</th></tr>
<tr><td>N/A</td><td>N/A</td><td colspan="2">Students should understand the idea of an index and write out the meaning of each term.</td><td>Students should now be able to apply their understanding of like terms without the need to write out any terms.</td></tr>
<tr><td colspan="2">Example(s)</td><td colspan="3">Example(s) - These should be seen in books</td></tr>
<tr><td colspan="5">Write $12b^5 \div 2b^3$ as a single power of b.</td></tr>
<tr><td colspan="2">NOT APPLICABLE</td><td>Students should be encouraged to write the original problem as a fraction, and then split this fraction into two parts
$12b^5 \div 2b^3$
$= \frac{12 \times b^5}{2 \times b^3}$
$= \frac{12}{2} \times \frac{b^5}{b^3}$
Students at this stage should then write the full meaning of each index
$= \frac{12}{2} \times \frac{b \times b \times b \times b \times b}{b \times b \times b}$
Students should then simplify the fractions by cancelling/
$= \frac{\overset{6}{\cancel{12}}}{\underset{1}{\cancel{2}}} \times \frac{b \times b \times \cancel{b} \times \cancel{b} \times \cancel{b}}{\cancel{b} \times \cancel{b} \times \cancel{b}}$
Students should then state their final answer
$= \frac{6}{1} \times b^2$
$= 6b^2$</td><td>Students should begin the second part of this stage in the same way as previous.
$12b^5 \div 2b^3$
$= \frac{12 \times b^5}{2 \times b^3}$
$= \frac{12}{2} \times \frac{b^5}{b^3}$
Students should then apply the law of index explicitly.
$= 6 \times b^{5-3}$
$= 6b^2$</td><td>Students should be able to state the final answer to the question without the need for any working.
$12b^5 \div 2b^3$
$= 6b^2$</td></tr>
</table>

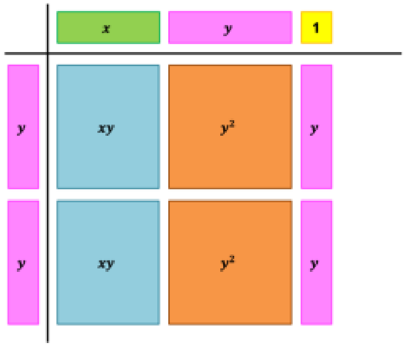
Power-on-Power Law

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**		Stage 4 **Abstract**
N/A	N/A	Students should understand the idea of an index and write out the meaning of each term.		Students should now be able to apply their understanding of like terms without the need to write out any terms.
Example(s)		**Example(s) - These should be seen in books**		
Write (2b⁴)³ as a single power of b.				
NOT APPLICABLE		$(4x^2)^3$ $= \underline{4x^2} \times \underline{4x^2} \times \underline{4x^2}$ $= \underline{4} \times \underline{4} \times \underline{4} \times \underline{x \times x} \times \underline{x \times x} \times \underline{x \times x}$ $= 64 \times x^6$ $= 64x^6$	$(4x^2)^3$ $= 4^3 \times (x^2)^3$ $= 64 \times x^{2\times3}$ $= 64x^6$	Students should be able to state the final answer to the question without the need for any working. $(4x^2)^3$ $= 64x^6$

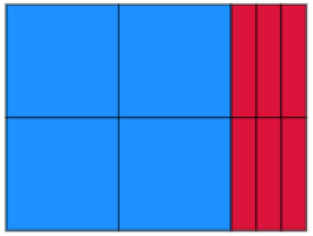
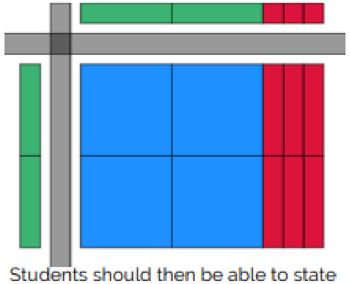
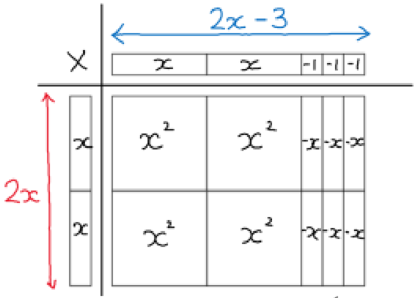
EXPANDING SINGLE BRACKETS

<table>
<tr><td>Stage 1
Physical</td><td>Stage 2
Pictorial</td><td>Stage 3
Semi-Abstract</td><td>Stage 4
Abstract</td></tr>
<tr><td>Students should be able to use algebra tiles to represent an area model.</td><td>Students should be able to link expanding brackets to the areas of rectangles. This should be represented in an adapted grid method that illustrates different variables as different sizes.</td><td>Students should extend their knowledge in the Concrete stage to remove the idea of lengths whilst still associating the working to area if required</td><td>Students should not need to use a grid, students at this stage should understand that each term inside the bracket must be multiplied by the term outside.</td></tr>
<tr><td>Example(s)</td><td colspan="3">Example(s) - These should be seen in books</td></tr>
<tr><td colspan="4">Expand 2(2x – 1)</td></tr>
<tr><td>

Students should understand that means "two lots of ". Students should then see that there are 4 's and 2 's, so</td>
<td><table><tr><td>x</td><td>2x</td><td>-1</td></tr><tr><td>2</td><td>Area =
4x</td><td>Area =
-2</td></tr></table>
Students should then understand that the expansion is by looking inside the grid

Notice that the sign is always included even if positive to emphasise the multiplication of a positive or negative number.</td>
<td>Students should then be able to expand a single bracket without the need to associate the size of each segment to the value.
<table><tr><td>x</td><td>2x</td><td>−1</td></tr><tr><td>2</td><td>4x</td><td>−2</td></tr></table>
$= 4x - 2$</td>
<td>$2(2x - 1)$
$= 4x - 2$</td></tr>
<tr><td colspan="4">Expand 2y(x + y + 1)</td></tr>
<tr><td>
</td>
<td><table><tr><td>X</td><td>x</td><td>+y</td><td>+1</td></tr><tr><td>2y</td><td>Area =
2xy</td><td>Area =
+2y²</td><td>Area=
+2y</td></tr></table>
$= 2xy + 2y^2 + 2y$</td>
<td><table><tr><td>x</td><td>x</td><td>+y</td><td>+1</td></tr><tr><td>2y</td><td>2xy</td><td>+2y²</td><td>+2y</td></tr></table>
$= 2xy + 2y^2 + 2y$</td>
<td>$2y(x + y + 1)$
$= 2xy + 2y^2 + 2y$</td></tr>
</table>

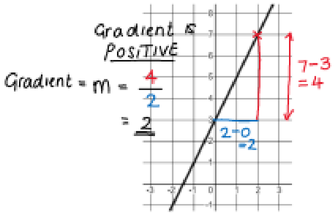
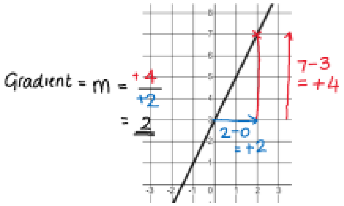
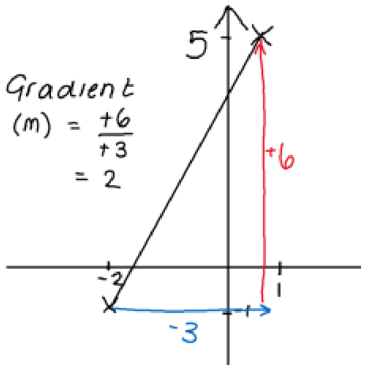
FACTORISING SINGLE BRACKETS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to use **algebra tiles** to represent an area model.	Students should be able to link factorising to the areas of rectangles. This should be represented in an adapted grid method that illustrates different variables as different sizes.	Students should extend their knowledge in the Concrete stage to remove the idea of lengths whilst still associating the working to area if required	Students should not need to use a grid, students at this stage should be able to determine the HCF of the terms and determine the final answer without working.
Example(s)	**Example(s) - These should be seen in books**		
Factorise $4x^2 - 6x$			
Students should arrange Algebra tiles in such a way that they form a perfect rectangle.  Students should then understand from expanding that the divisor goes outside of the bracket (the height of the diagram) and the quotient goes inside of the bracket (the width of the diagram. Students should use additional tiles to identify this. It may be useful for students to draw a line along the left side and top to help distinguish the tiles.  Students should then be able to state $4x^2 - 6x = 2x(2x - 3)$	Students should draw the tiles in their books, using the same ideas as the Physical Step. At this stage their sizes should be the same as the physical tiles.  $4x^2 - 6x = 2x(2x - 3)$	Students should represent the expression in a grid, but without the need for it to be a scaled diagram. $\begin{array}{c\|c} \times & 2x - 3 \\ \hline 2x & 4x^2 - 6x \end{array}$ $= 2x(2x - 3)$	Students should be able to state their factorised answer without the need to use a diagram. $4x^2 - 6x$ $= 2x(2x - 3)$

SOLVING INEQUALITIES

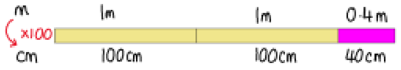
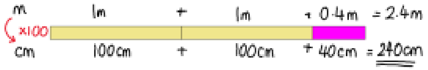
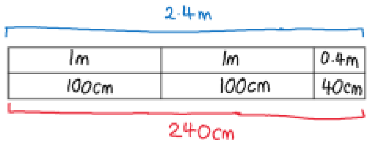
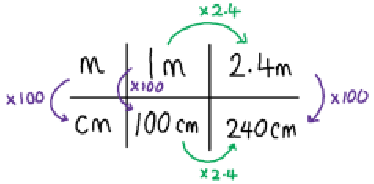
<table>
<tr><td>GENERAL TEACHING & LEARNING POINTS</td><td colspan="3">- TBA</td></tr>
<tr><td>Stage 1
Physical</td><td>Stage 2
Pictorial</td><td>Stage 3
Semi-Abstract</td><td>Stage 4
Abstract</td></tr>
<tr><td>N/A</td><td>N/A</td><td>Students should split the inequality into two parts, solving each separately.</td><td>Students should be able to apply the same operation to all three parts of the inequality, therefore removing the need to split the inequality into two parts.</td></tr>
<tr><td>Example(s)</td><td colspan="3">Example(s) - These should be seen in books</td></tr>
<tr><td colspan="4">Solve the inequality $-3 \leq 2x + 5 < 7$. Represent your solution on a number line.</td></tr>
<tr><td>NOT APPLICABLE</td><td>NOT APPLICABLE</td><td>Students should use their solving equations skills to be able to perform similar operations.
NOTE: The inequality symbols should be left in throughout and not replaced with an - sign.
$\underbrace{-3 \le \overbrace{2x+5}}\overbrace{< 7}$
$\begin{array}{lcr|lcr} & -3 \le 2x+5 & & & 2x+5<7 & \\ -5 & -8 \le 2x & -5 & -5 & 2x<2 & -5 \\ \div 2 & -4 \le x & \div 2 & \div 2 & x<1 & \div 2 \end{array}$
Students should then combine their answers to form the final solution set
$-4 \leq x < 1$
Students should then represent their answer on a number line as specified in the question.
 </td><td>At the Abstract Stage, students should not need to separate the two parts of the inequality, instead they should apply the inverse operations to all three parts of the inequality equation.
$\begin{array}{lcccr} & -3 \le & 2x+5 & < 7 & \\ -5 & & \downarrow -5 & & -5 \\ & -8 \le & 2x & < 2 & \\ \div 2 & & \downarrow \div 2 & & \div 2 \\ & -4 \le & x & < 1 & \end{array}$
Students should then represent their answer on a number line as specified in the question.
 </td></tr>
</table>

CALCULATING THE GRADIENT OF A STRAIGHT LINE

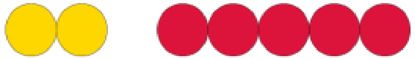
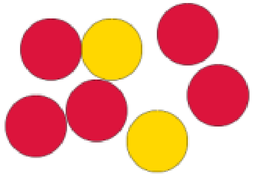
GENERAL TEACHING & LEARNING POINTS	• TBA		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to calculate the gradient from a graph.	Students should be able to calculate the gradient from two points and drawing a sketch.	Students should be able to calculate the gradient from two points and drawing a sketch.
Example(s)	**Example(s) - These should be seen in books**		
	Determine the gradient of the straight line below	Calculate the gradient of the line that passes through (-2,-1) and (1,5)	
NOT APPLICABLE	At this stage students work out if the gradient is positive or negative, work out the absolute value of the gradient and add in the – sign afterwards if needed.  Higher attaining students will be able reduce this to:  	At this stage the problem has been prepresented as a diagram, similar to that of the Pictorial Stage. This has been represented on an axis, however, this is not necessary. A simple sketch would suffice.  	At this stage students should use the $\frac{Change\ in\ y}{Change\ in\ x} = \frac{y_2-y_1}{x_2-x_1}$ formula. $(\underset{x_1}{-2}, \underset{y_1}{-1}), (\underset{x_2}{1}, \underset{y_2}{5})$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5 - {}^{-}1}{1 - {}^{-}2}$ $= \frac{6}{3}$ $= 2$

Ratio and Proportion

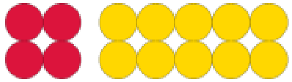
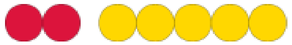
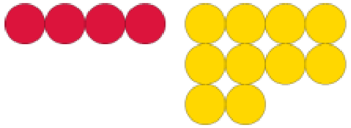
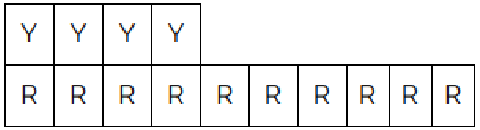
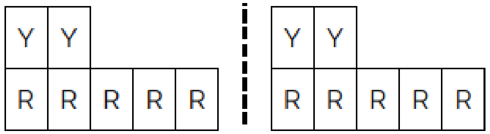
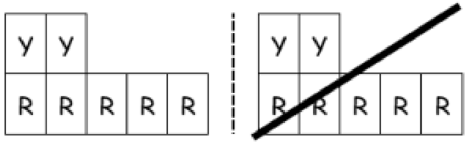
CONVERTING UNITS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students could use Cuisenaire Rods (base 10) to represent the original length.	Students should use a bar model to represent a similar idea to that of the Physical Stage.	Students should use a proportion table to scale up or down each quantity.	Students should be able to state the final conversion without any working.
Example(s)	**Example(s) - These should be seen in books**		
Convert 2.4m into cm.			
It will be beneficial if students use the manipulatives on top of a whiteboard for this stage. Students should represent the 2.4m using two '10' rods and one '4' rod.  Notice the scale factor has been indicated on the left hand side. Students should then sum above and below.  Students should then state their final answer. 2.4m = 240cm	Students should represent the problem as a bar model clearly equating each part of the bar model in both units, as shown below.  Notice, the diagram is roughly to scale (0.4m is smaller than the 1m), this emphasises that, when converted, the 0.4m cannot be more than 100cm. Students should then state their final answer. 2.4m = 240cm	Students should set the work out in a proportion table. **ALL students** should be able to perform the conversion in two different ways, shown in green and purple below.  Students should then state their final answer. 2.4m = 240cm	Students will be able to state the conversion immediately. 2.4m = 240cm

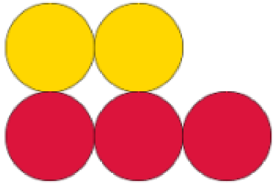
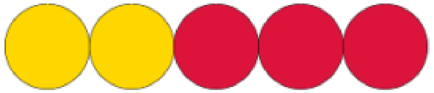
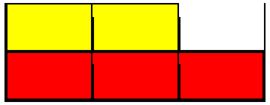
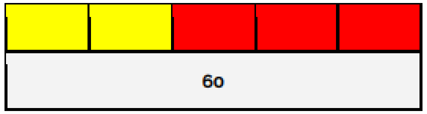
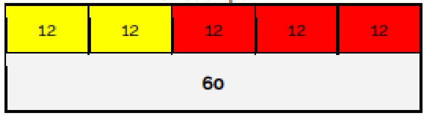
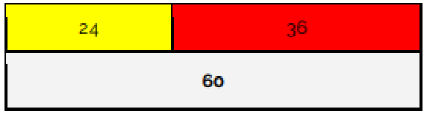
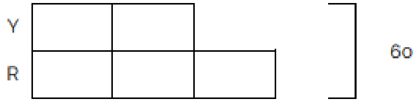
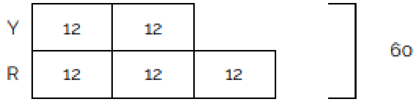
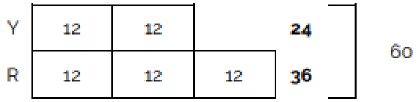
WRITING RATIOS AS A FRACTIONS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using counters.	Students should be able to represent the information using a bar model.	Students should be able to use a single bar model to aid their understanding	Students should be able to write the fraction without the need for a diagram.
Example(s)	**Example(s) - These should be seen in books**		
The ratio of yellow to red counters is 2:5. What fraction of the counters is yellow?			
Students should be able to represent the problem as below.  Notice that this clearly shows the different counters separately. Students should then physically move the counters together, this is important so that students understand that in total there are 7 parts.  Students should then apply their understanding of what a fraction is to identify that 5 out of the 7 counters are red. Students should clearly write this. 2 out of 7 are yellow $\frac{2}{7}$ are yellow	Students should represent this as a bar model, with colour side-by-side. Y Y : R R R R R Students should then combine each side into a single bar to represent the total number of parts. Y Y R R R R R Students should then add the fraction of each colour underneath their representation. $\underbrace{Y\ Y}_{\frac{2}{7}}\underbrace{R\ R\ R\ R\ R}_{\frac{5}{7}}$ $\frac{2}{7}$ are yellow	Students should understand the idea of a ratio by this stage. Students should therefore be able to draw the diagram below: Y Y R R R R R Students should then be able to state the appropriate fraction from this diagram alone. $\frac{2}{7}$ are yellow	Students should be able to represent the problem using a table. Consistency is important here as this layout will be used regularly during sharing in a ratio. Yellow : Red 2 : 5 Total Parts 2 + 5 = 7 $\frac{2}{7}$ are yellow

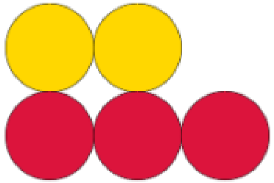
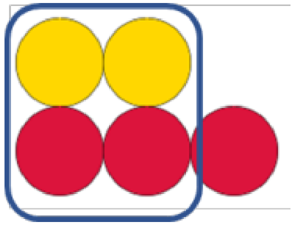
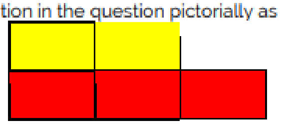
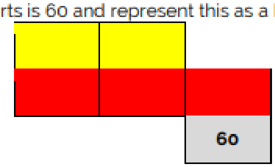
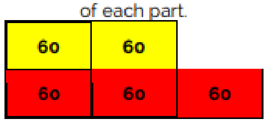
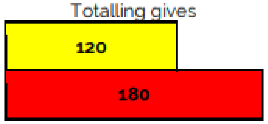
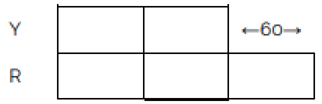
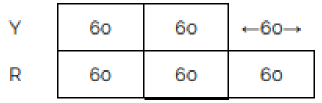
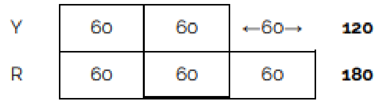
SIMPLIFYING RATIO

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.
Example(s)	**Example(s) - These should be seen in books**		
Simplify the ratio 4:10.			
Students should be able to represent the ratio using counters.  Students should then attempt to re-arrange the counters into the same number of rows, two in this case.  Students should apply their understanding of what a ratio is to remove all rows of counters except the first.  Students should then be able to state that 4:10 can be simplified to 2:5. **Discussion Point** Students should be able to confidently explain why the counters cannot be arranged as below 	Students should represent this as a 'comparative' bar model, with colour on top of each other  Students should then recognise that the diagram would be equal if two yellow parts were moved as shown:  Students should apply their understanding of what a ratio is to remove all but one group and the ratio remains equal.  R : Y = 2 : 5	Students should represent the ratio in a single line. Y Y Y Y : R R R R R R R R R R Students should then be able to identify that each side of the ratio could be grouped into 2s. Higher attaining students could identify that this is the highest common factor of the two parts. Students should circle these groups.  Students should then understand the ratio of the original number of each colour is the same ratio as the number of groups. R : Y = 2 : 5	Students should represent the ratio in a table. Students should clearly show that the division operations need to be applied to both sides of the ratio to keep them equivalent. R : Y 4 : 10 ÷2 2 : 5 ÷2

SHARING IN A RATIO – SHARING TOTAL

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.
Example(s)	**Example(s) - These should be seen in books**		
There are 60 counters in a box. They are either yellow or red. The ratio of yellow counters to red counters is 2:3. How many of each colour are in the box?			
Students should be able to represent the problem as below.  Students should understand that the total value of all five parts is 60 and all parts represent the same amount, i.e. the manipulative below represents 60. Students should then identify the value of one cube:  Students should then understand since there are 2 yellow and 3 red blocks this represents: Yellow: 12 + 12 = 24 Red: 12 + 12 + 12 = 36 [Notice that this is represented as repeated addition and not multiplication] So there are 24 yellow and 36 red.	Students should be able to represent the information in the question pictorially as a 'comparative' bar model as below:  [Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later] Students should understand that the total value of all five parts is 60 and represent this as a 'cumulative' bar model.  Students should then be able to identify the value of each part.  Students will then see the number of each colour.  So there are 24 yellow and 36 red.	Students should be able to represent the information in the question pictorially, including the total number, as below:  Students should then know to find the value of each part is the total divided by the number of parts. Students should then annotate the value of each part on their diagram:  Totalling each colour  So there are 24 yellow and 36 red.	Students should be able to identify the <u>total</u> number of parts, then identify the scale factor to multiply to make the total parts to the total number of counters in this case. ***Note: Parts in <u>black</u> are step one, parts in <u>blue</u> are step two.*** Yellow : Red | Total Parts 2 : 3 | 5 X12 X12 X12 24 : 36 | 60 Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question. Y:R = **24:60** OR Yellow = **24 counters** Red = **36 counters**

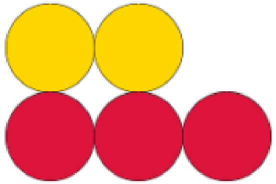
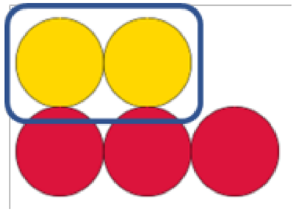
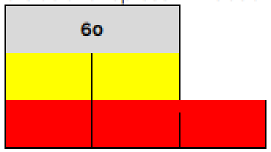
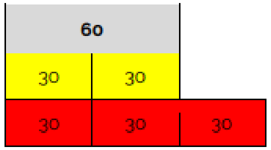
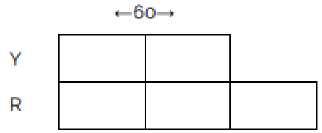
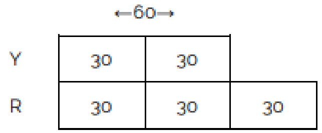
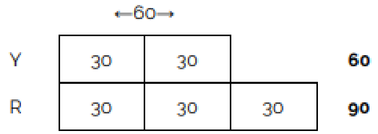
SHARING IN A RATIO - DIFFERENCES

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.
Example(s)	**Example(s) - These should be seen in books**		
They are either yellow or red counters in a box. The ratio of yellow counters to red counters is 2:3. There are 60 **more** red counters than yellow. How many yellow counters and red counters were in the box?			
Students should be able to represent the problem as below.  Students should understand that the four blocks circled have equal parts and the extra red block represents the 60 extra red counters.  So each block represents 60 Students should then understand since there are 2 yellow and 3 red blocks this represents: Yellow: 60 + 60 = 120 Red: 60 + 60 + 60 = 180 So there are 120 yellow and 180 red.	Students should be able to represent the information in the question pictorially as below:  [Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later] Students should understand that the total value of all five parts is 60 and represent this as a bar model.  Students should then be able to identify the value of each part.  Totalling gives  So there are 120 yellow and 180 red.	Students should be able to represent the information in the question pictorially, including the difference, as below:  Students should then understand each part must represent the same value. Students should annotate their diagram.  Students should then be able to total each part.  So there are 120 yellow and 180 red.	Students should be able to identify the <u>difference</u> in the number of parts, then divide to determine the amount represented by 1 part. ***Note: Parts in <u>black</u> are step one, parts in <u>blue</u> are step two.*** Yellow : Red | Parts Difference 2 : 3 | 1 X60 120 X60 : 180 | 60 X60 Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question. Y:R = **120:180** OR Yellow = **120 counters** Red = **180 counters**

SHARING IN A RATIO – GIVEN ONE AMOUNT

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.
Example(s)	**Example(s) - These should be seen in books**		

They are either yellow or red counters in a box. The ratio of yellow counters to red counters is 2:3. There are 60 yellow counters. How many counters are in the box in **total**?

Students should be able to represent the problem as below.  Students should understand that the yellow blocks represent 60.  So each block represents 30. Students should then understand that the 3 red blocks this represents: Red: 30 + 30 + 30 = 90 So there are 60 + 90 = 150 in total.	Students should be able to represent the information in the question pictorially as below:  [Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later] Students should understand that the total value of all five parts is 60 and represent this as a bar model.  Students should then be able to identify the value of each part.  Totalling gives  **60+90 = 150 counters in total**	Students should be able to represent the information in the question pictorially, including the total yellow counters., as below:  Students should then understand each part must represent the same value. Students should annotate their diagram.  Students should then be able to total each part and the overall total.  **60+90 = 150 counters in total**	Students should be able to identify the number of parts represented by the given quantity, then divide to determine the amount represented by 1 part. ***Note: Parts in black are step one, parts in blue are step two.*** Yellow : Red | Total Parts 2 : 3 | 5 X30 X30 X30 60 : 90 | 150 Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question. Y:R = 60:90 Total = 60 + 90 = **150**

COMPOUND MEASURES

<table>
<tr>
<td>GENERAL TEACHING & LEARNING POINTS</td>
<td colspan="3">

- Students should have a secure understanding of units and what these units mean. E.g. mph is "how many miles something travels per hour" or Pressure "How many Newtons per every 1 cm^{2}"
- Students should apply their understanding of proportion when working with Compound Measures
- Try to avoid using formula triangles with Compound Measures as these do not promote teaching for understanding.

</td>
</tr>
<tr>
<td>Stage 1
Physical</td>
<td>Stage 2
Pictorial</td>
<td>Stage 3
Semi-Abstract</td>
<td>Stage 4
Abstract</td>
</tr>
<tr>
<td>N/A</td>
<td>Students should use a bar model to represent the problem.</td>
<td>Students should use a proportion table to scale up or down each quantity.</td>
<td>Students should confidently state and use the Compound Measures formulae without the need for an alternative representation.</td>
</tr>
<tr>
<td>Example(s)</td>
<td colspan="3">Example(s) - These should be seen in books</td>
</tr>
<tr>
<td colspan="4">A car travels at a speed of 45 mph for 20 minutes. How far does it travel in this time?</td>
</tr>
<tr>
<td>NOT APPLICABLE</td>
<td>
Students should represent this as a bar model.

Notice that the distance travelled in 1 hour is represented using equal length bars.

<table>
<tr><td>45 miles</td></tr>
<tr><td>1 hour</td></tr>
</table>

Students should then understand that 20 minutes is one third of an hour and therefore this needs to be split into three equal parts.

Students should then use proportion to calculate the distance travelled in this time.

<table>
<tr><td>15 miles</td><td>15 miles</td><td>15 miles</td></tr>
<tr><td colspan="3">45 miles</td></tr>
<tr><td colspan="3">1 hour</td></tr>
<tr><td>20 mins</td><td>20 mins</td><td>20 mins</td></tr>
</table>

Students should then be able to identify that in 20 minutes the car travelled 15 miles.
</td>
<td>
At this stage students should use a proportion table to determine the unknown quantity.

÷3

<table>
<tr><td>Distance</td><td>45 miles</td><td>15 miles</td></tr>
<tr><td>Time</td><td>1 hour</td><td>$\frac{1}{3}$ hour</td></tr>
</table>

÷3
</td>
<td>
Students should use and substitute into the formula.

Avoid using the formula triangles, however, this should be the only stage at which then could be used if students are unable to understand the basic concepts after significant time has been spent.

Students should clearly state the formula they are using (in its original form) followed by any re-arrangement as a separate stage of working.

Students should state the value of quantities in the question, with particular emphasis on units.

$S = 45$ mph $\quad t = \frac{1}{3}$ hour

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

Distance = Speed x Time

Distance = $45 \times \frac{1}{3}$ = **15 miles**
</td>
</tr>
</table>

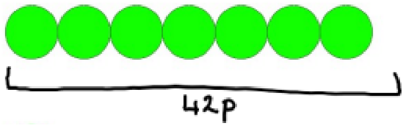
DIRECT PROPORTION

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent this problem using counters or multi-link cubes.	Students should be able to represent the information using a bar model in their books.	Students should use a direct proportion table to be able to represent and solve the problem	Students should be able to use direct proportion using the same format as simplifying a ratio.
Example(s)	**Example(s) - These should be seen in books**		

7 pens cost 42p. What is the cost of 1 pen?

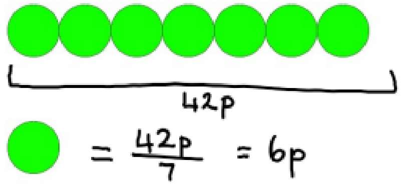
Stage 1

Students should represent the problem by using 7 counters or cubes to represent the 7 pens.



Students should then be able to work out the value of 1 pen.

Higher attaining students will be able to spot that, for other problems, that it isn't always necessary to find the value of 1 part first. E.g. 8 pens cost 80p work out the cost of 2 pens. Students will be able to divide by 4 in one step rather than dividing by 8 then multiplying by 2.

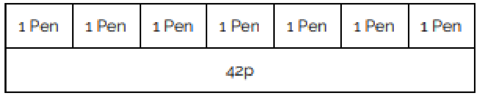


Students should then clearly answer the question.

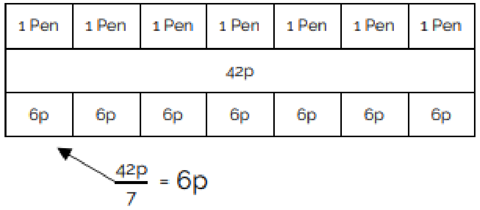
1 pen = 6p

Stage 2

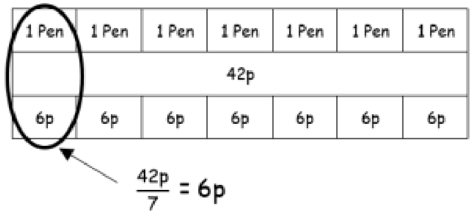
Students should draw the diagram below in their books.



Students should then clearly show that they have divided the 42p into 7 parts by adding this to their diagram and showing the division calculation.



Students could then circle the part of their diagram that gives the value of 1 pen.

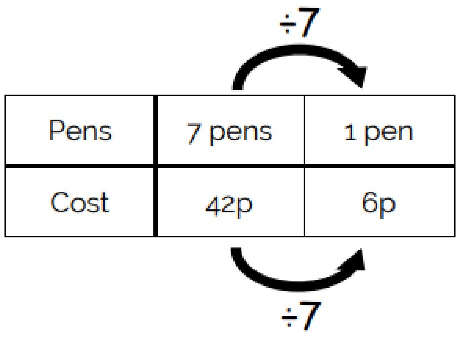


Students should then clearly answer the question.

1 pen = 6p

Stage 3

At this stage students should use a direct proportion table to determine the unknown quantity.



Students should then clearly answer the question.

1 pen = 6p

Stage 4

Students should represent the problem in a table.

Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.

Pens : Cost
7 pens : 42p
÷7 1 pen : 6p ÷7

Students should then clearly answer the question.

1 pen = 6p

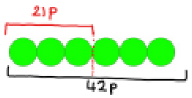
BEST BUY PROBLEMS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent this problem using counters or multi-link cubes.	Students should be able to represent the information using a bar model in their books.	Students should use a direct proportion table to be able to represent and solve the problem	Students should be able to use direct proportion using the same format as simplifying a ratio.
Example(s)	**Example(s) - These should be seen in books**		

Shop A sells 6 pens for 42p. Shop B sells 9 pens for 72p.
Bella wants to buy some pens from one of the shops. Which shop should Bella buy the pens from?

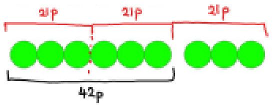
Students should represent the problem by using 7 counters or cubes to represent the 7 pens.

Students should then be able to work out the value of 3 pens.

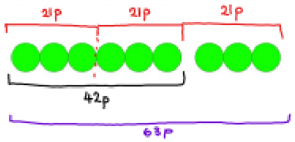


Students could use a unitary approach here, however, students should be encouraged to be efficient with their method and therefore since 9 is a multiple of 3 this would be more effective.

Students should then add in the additional counters to their diagram in order to work out the cost of an equivalent number of pens as Shop B but from Shop A.



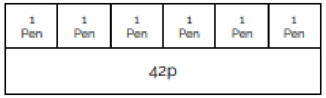
Students should then work out the cost from Shop A

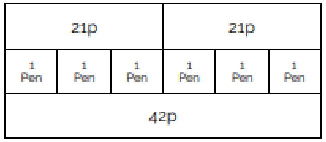


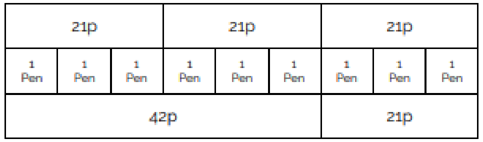
Students should then clearly answer the question.

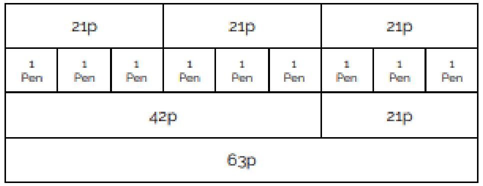
Bella should buy from Shop A

Students should draw similar diagrams to the Physical Stage below in their books, this time using a bar model







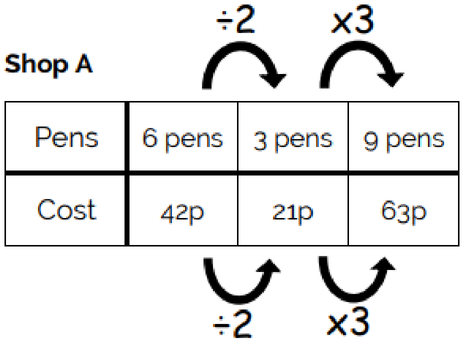


Students should then clearly answer the question.

Bella should buy from Shop A

At this stage students should use a proportion table to determine the cost of pens in Shop A.

Shop A



Pens	6 pens	3 pens	9 pens
Cost	42p	21p	63p

Bella should buy from Shop A

Students should represent the problem in a table.

Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.

Shop A

	Pens	:	Cost	
	6 pens	:	42p	
÷2	3 pens	:	21p	÷2
x3	9 pens	:	63p	x3

Students should then clearly answer the question.

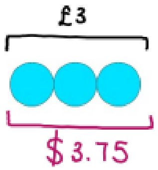
Bella should buy from Shop A

EXCHANGE RATES

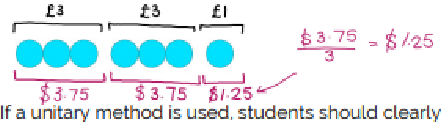
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent this problem using counters or multi-link cubes.	Students should be able to represent the information using a bar model in their books.	Students should use a direct proportion table to be able to represent and solve the problem	Students should be able to use direct proportion using the same format as simplifying a ratio.
Example(s)	**Example(s) - These should be seen in books**		

The exchange rate from Pounds (£) into US Dollars ($) is £3 = $3.75.
Calculate how many Dollars ($) are equivalent to £7.

Students should be able to represent the initial problem and information with a manipulative.

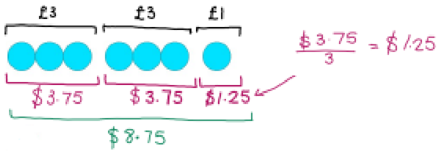


Students should then be able to scale up the given quantity, by either using a unitary method or by understanding that £7 is 2 x £3 + £1, as shown:



If a unitary method is used, students should clearly show how they obtained the value of a single unit.

Students should then be able to add up the corresponding amounts and give a final answer.



£7 = $8.25

Students should draw similar diagrams to the Physical Stage below in their books, this time using a bar model.

£3
$3.75

Students should clearly show how they are scaling up the value.

£3	£3	£1
$3.75	$3.75	$1.25

$$£1 = \frac{\$3.75}{3} = \$1.25$$

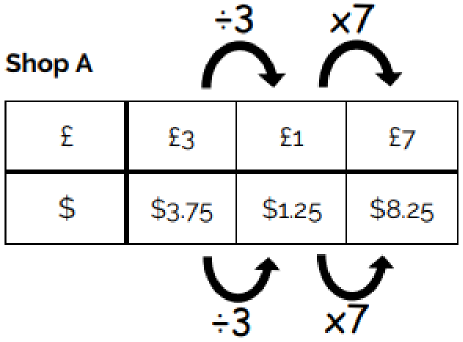
Students should then show the total value and clearly state their final answer.

£3	£3	£1
$3.75	$3.75	$1.25
$8.25		

£7 = $8.25

At this stage students should use a proportion table to determine the cost of pens in Shop A.

Shop A



£	£3	£1	£7
$	$3.75	$1.25	$8.25

Students should then clearly state their final answer.

£7 = $8.25

Students should represent the problem in a table.

Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.

	£	:	$	
	£3	:	$3.75	
÷3	£1	:	$1.25	÷3
x7	£7	:	$8.25	x7

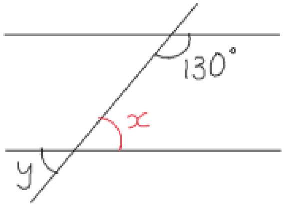
Students should then clearly state their final answer.

£7 = $8.25

Geometry

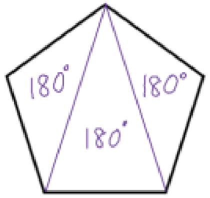
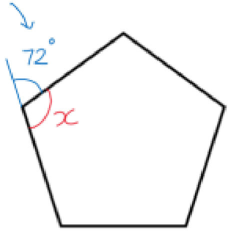
ANGLES – STRAIGHT LINES AND POINTS

<table>
<tr><td>GENERAL TEACHING & LEARNING POINTS</td><td colspan="3">- Students should constantly be made to justify their answers.
- Consistent terminology is essential. The terminology to be used is below.</td></tr>
<tr><td>VOCABULARY</td><td colspan="3">Corresponding angles are equal.
Alternate angles are equal.
Co-interior angles add up to 180˚
Vertically opposite angles are equal
Angles on a straight line add up to 180˚
Angles around a point add up to 360˚</td></tr>
<tr><td>Stage 1
Physical</td><td>Stage 2
Pictorial</td><td>Stage 3
Semi-Abstract</td><td>Stage 4
Abstract</td></tr>
<tr><td>Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.</td><td colspan="3">Since this is a pictorial topic, these stages are combined.
Students should form and solve an equation given angle facts.</td></tr>
<tr><td>Example(s)</td><td colspan="3">Example(s) - These should be seen in books</td></tr>
<tr><td rowspan="2">Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.</td><td colspan="3">Students should form and solve an algebraic equation in order to determine the missing angle.
Work out the size of angle x.


$x + 90° + 30° = 180°$
$x + 120° = 180°$
-120° -120°
$x = 60°$</td></tr>
<tr><td colspan="3">Work out the size of angle y. Give reasons at each stage of your working.


Working | Reasons
$x + 130° = 180°$ | Co-interior Angles add up to 180°
$x = 50°$
$y = 50°$ | Vertically opposite angles are equal
Students could be encouraged to split their page in half with one column as the working and the second for the reasons.</td></tr>
</table>

INTERIOR AND EXTERIOR ANGLES

GENERAL TEACHING & LEARNING POINTS	- Students should not purely be taught a formula, students should understand that the volume links to the number of layers of a shape - Students should then understand that the volume of a prism (and cylinder) is the Area of the Cross-Section x Length			
Stage 1 **Physical**		Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.		Students should determine the interior and exterior angles of a polygon by splitting the shape into triangles.	Students should understand the link between interior and exterior angles.	Students should understand and apply the formula $(n-2) \times 180°$
Example(s)		**Example(s) - These should be seen in books**		
		Determine the sum of the interior angles of a regular pentagon.		
Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.		Students should pick a particular point and separate the polygon into triangles.  Pentagon = 3 triangles $= 3 \times 180°$ $= 540°$	Students should understand that interior and exterior angles sum to 180˚ and that the sum of the exterior angles of any polygon sum to 360˚. $\frac{360°}{5} = 72°$  $-72° \; (\; x + 72° = 180° \;) \; -72°$ $x = 108°$ Sum of interior Angles = $108° \times 5 = 540°$	Pentagon = 5 sides Sum of interior Angles $= (n-2) \times 180°$ $= (5-2) \times 180°$ $= 3 \times 180°$ $= 540°$

AREA OF POLYGONS AND CIRCLES

GENERAL TEACHING & LEARNING POINTS

- Students should be taught the areas of polygons in the following order:

Rectangles → Parallelograms → Triangles → Trapeziums → Circles

- Students should NOT be merely taught a formula. Students should **understand** that each formula can be determined from the area of a parallelogram or rectangle

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students could use physical versions of a rectangle or parallelogram to determine the formulae for the area of each shape.	Students should be able calculate the area of a shape, including compound shapes. Students should clearly state any formula and any values given in the questions. For answers involving π, students should always state the exact answer prior to using any rounding.		

Example(s)

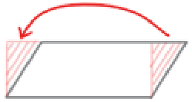
Rectangle

A = b x h OR A = bh

Parallelogram

A = b x h OR A = bh

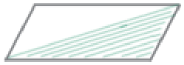
Students should move part of the parallelogram to the opposite side to form a rectangle.



Parallelogram

$A = \frac{b \times h}{2}$ OR $A = \frac{bh}{2}$

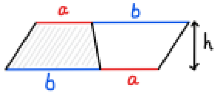
Students should split a parallelogram in half to form a triangle.



Trapezium

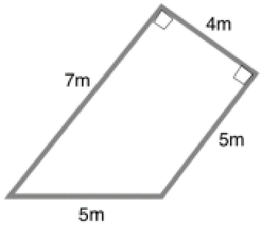
$A = \frac{a+b}{2} \times h$

Students should rotate two trapezia to identify the area is half a parallelogram with side length (a+b)



Example(s) - These should be seen in books

Calculate the area of this trapezium.



Students should clearly state the formula to be used.

$A = \frac{a+b}{2} \times h$

Students should state the required information from the diagram.

a = 5m b = 7m h = 4m

Students should then substitute into the formula

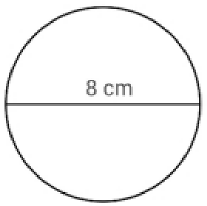
$A = \frac{5+7}{2} \times 4$

A = 6 x 4

Students should ensure that the units are included in the final answer.

$A = 24m^2$

Calculate the area of this circle.



Students should clearly state the formula to be used.

$A = \pi r^2$

Students should state the required information from the diagram.

d = 8cm so r = 4cm

Students should then substitute into the formula

$A = \pi \times 4^2$

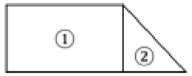
Students must include the answer in terms of π answer

A = 16π

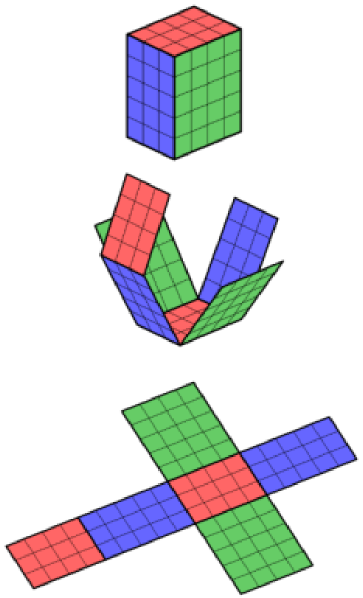
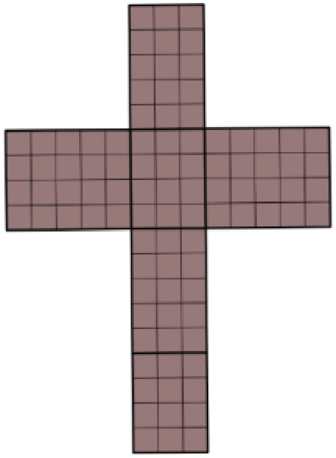
Students should ensure that the units are included in the final answer.

$A = 50.27cm^2$

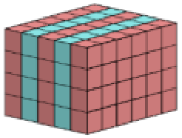
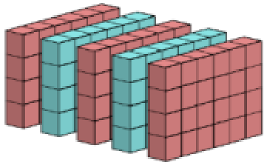
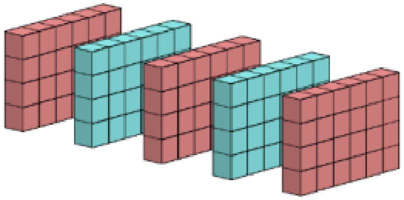
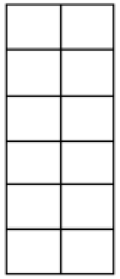
When working with compound shapes, students should clearly label each part of the shape.
Students should then calculate each area separately as shown above.



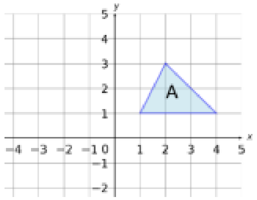
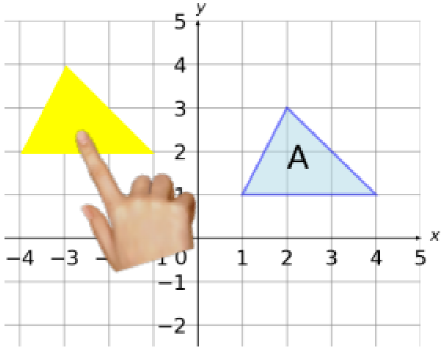
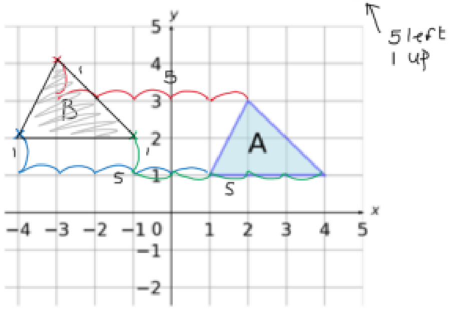
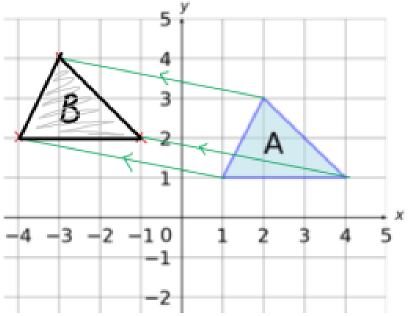
SURFACE AREA

GENERAL TEACHING & LEARNING POINTS	• Students should be encouraged to understand what is my meant by the surface area by investigating the nets of shapes. • Students should be able to find the surface area of cubes, cuboids, prisms and cylinders.		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should use the 3D Geometric Shapes to understand the different faces.	Students should use a net to find the surface area.	Students should list each of the faces then total.	Students should identify repeated faces.
Example(s)	**Example(s) - These should be seen in books**		
A cuboid has a length of 3cm, a width of 4cm and a height of 5cm. Work out the surface area of the shape.			
Students should use the 3D Geometric Shapes to understand the different faces. The Maths Pad interactive tool could be used to help understanding.  Students will then be able to count the squares to find the Surface Area.	Students should draw a net of the cuboid using the squares in their books.  Surface Area = 94cm²	Students should list all of the sides and calculate the areas of each. Students should then total these. Front 5 x 4 = 20 Back 5 x 4 = 20 Top 4 x 3 = 12 Bottom 4 x 3 = 12 Left 3 x 5 = 15 Right 3 x 5 = 15 Surface Area = 94cm²	Students should identify the faces that have equal area. Students should then total these. Front 5 x 4 = 20 Top 4 x 3 = 12 Left 3 x 5 = 15 Total = 47 Surface Area = 2 x 47 = 94cm²

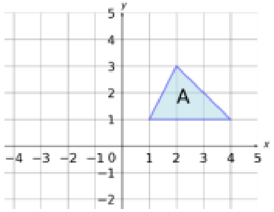
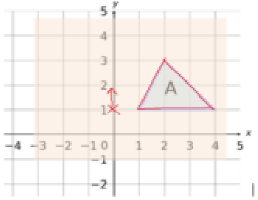
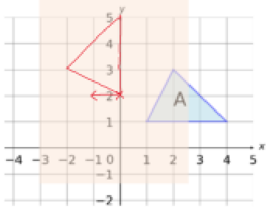
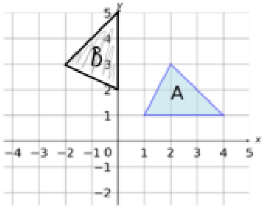
VOLUME

GENERAL TEACHING & LEARNING POINTS	- Students should not purely be taught a formula, students should understand that the volume links to the number of layers of a shape - Students should then understand that the volume of a prism (and cylinder) is the Area of the Cross-Section x Length		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should use the 3D Geometric Shapes to understand the different faces.	Students should draw the cross section, find the area and multiply by the length.	Students should find the area of the cross section and multiply by the length.	Students should state and use a formula.
Example(s)	**Example(s) - These should be seen in books**		
	A cuboid has a length of 4cm, a width of 2cm and a height of 6cm. Work out the volume of the shape.		
Students should use the 3D Geometric Shapes to understand the different faces and the idea of a cross-section. The Maths Pad interactive tool could be used to help understanding. Students should understand that the volume is the area of the cross-section multiplied by the number of layers.   	 Cross Section A = bh A = 2 x 6 A = 12cm^2 Volume = Area of Cross-Section X Length V = 12 x 4 = 48cm^3	Volume = Area of Cross-Section X Length Area of CS = 2 x 6 = 12cm^2 V = 12 x 4 = 48cm^3	V = lwh V = 4 x 2 x 6 = 48cm^3

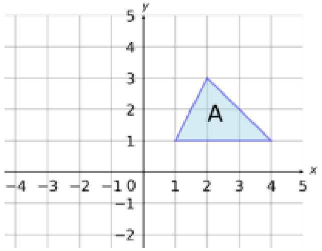
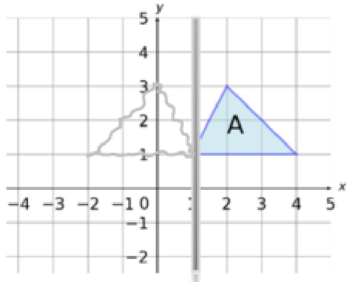
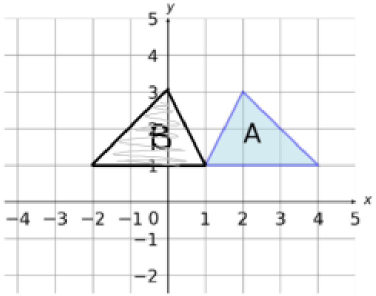
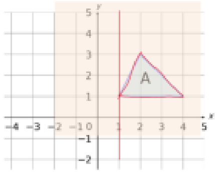
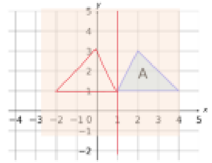
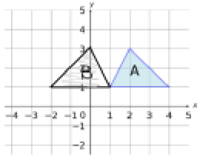
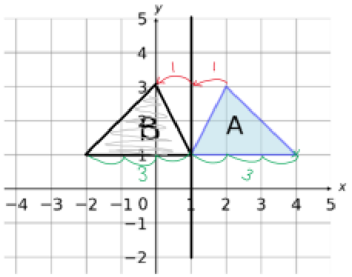
TRANSFORMATIONS

GENERAL TEACHING & LEARNING POINTS	- Students should understand the idea of a vector and writing and describing vectors prior to performing transformations - Students should understand the meaning of the terms Object and Image, and this terminology should be used throughout the unit.		
Translations			
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should use a physical, cut out, shape to move the object to the image's location.	Students should draw on their grid how the shape will move. Students should do this from each point.	Students should draw the vector that is to be applied to each point.	Students should understand that a vector has a fixed origin and therefore the points can be determined through vector arithmetic.
Example(s)	**Example(s) - These should be seen in books**		
Translate shape A (shown on the right) along the vector $\binom{-5}{1}$			
	Translate shape A along the vector $\binom{-5}{1}$ ↔ ↕ 	Translate shape A along the vector $\binom{-5}{1}$ 	$\binom{1}{1} + \binom{-5}{1} = \binom{-4}{2}$ $\binom{4}{1} + \binom{-5}{1} = \binom{-1}{2}$ $\binom{2}{3} + \binom{-5}{1} = \binom{-3}{4}$

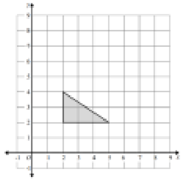
Reflections

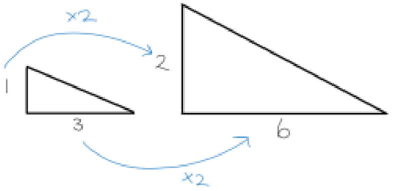
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Due to the pictorial nature of reflections, the process outlined below satisfies the Physical, Pictorial and Semi-Abstract stages.			
Example(s)	**Example(s) - These should be seen in books**		
Rotate shape A 90° anti-clockwise about the point (0, 1)			
Students should clearly mark the centre of rotation and draw an upward arrow from it. Students should trace over the shape, as shown below.  Students should then rotate the tracing paper so that the arrow points in the required direction after the rotation and then trace over the shape.  Students should ensure that their final image is drawn clearly with a ruler. 			*The full Abstract Stage of rotations requires knowledge of **Matrices** which is not covered at GCSE.*

Reflections

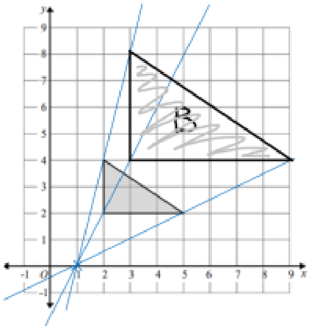
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should use a physical mirror to reflect the shape.	Students should use tracing paper to reflect the shape.	Students should draw the vector that is to be applied to each point.	Students should understand that a vector has a fixed origin and therefore the points can be determined through vector arithmetic.
Example(s)	**Example(s) - These should be seen in books**		
Reflect the shape A in the line x = 1.			
 Students should then accurately draw the image with a ruler. 	Students should use tracing paper to trace the shape **and** the mirror line.  Students should flip the tracing paper over and line up the mirror line.  Students should then accurately draw the image with a ruler. 	Students should be able to perform the reflection by determining how to get from each vertex of the shape and then performing the appropriate same step to get to the point on the image. 	*The full Abstract Stage of Reflections requires knowledge of **Matrices** which is not covered at GCSE.*

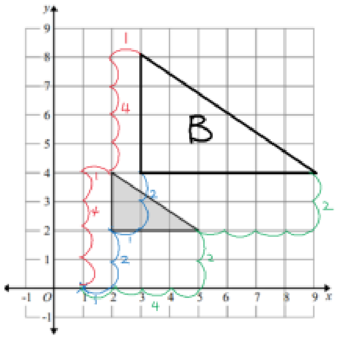
Enlargements

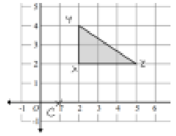
<table>
<tr><th>Stage 1
Physical</th><th>Stage 2
Pictorial</th><th>Stage 3
Semi-Abstract</th><th>Stage 4
Abstract</th></tr>
<tr><td>Students should be able to determine the dimensions of a shape after an enlargement.</td><td>Students should use a ray diagram to draw the enlargement</td><td>Students should determine the vector from the centre of rotation to each vertex, then perform that vector the required number of times.</td><td>Students should be able to multiply the vector by the SF of the enlargement.</td></tr>
<tr><td>Example(s)</td><td colspan="3">Example(s) - These should be seen in books</td></tr>
<tr><td colspan="3">Enlarge the shape on the right by a scale factor of 2, centred at (1, 0).</td><td></td></tr>
<tr>
<td>Students should be able to determine and draw the size of the enlarged shape.


</td>
<td>Students should draw straight lines through each vertex of the shape and the centre of enlargement.

Students should understand what dimensions the enlarged shape will have.


</td>
<td>Students should determine the translation from the centre of enlargement to each vertex of the shape and repeat this the same number of times as the scale factor.


</td>
<td>Students should label the centre of enlargement C and the vertices of the shape using different letters.

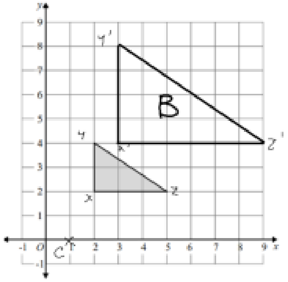


Students should then use scalar vector multiplication to determine the vector from the centre to that point on the image.

$\overrightarrow{CX} = \begin{pmatrix}1\\2\end{pmatrix}$ so $CX' = 2\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix}$

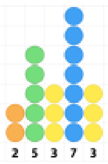
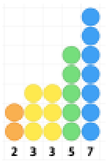
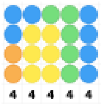
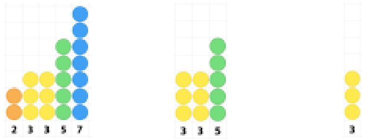
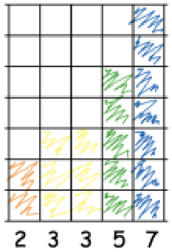
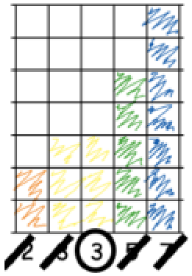
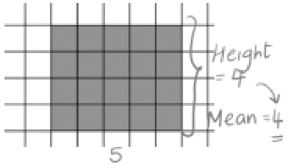
$\overrightarrow{CY} = \begin{pmatrix}1\\4\end{pmatrix}$ So $\overrightarrow{CY'} = 2\begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}2\\8\end{pmatrix}$

$\overrightarrow{CZ} = \begin{pmatrix}4\\2\end{pmatrix}$ so $\overrightarrow{CZ'} = 2\begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}8\\4\end{pmatrix}$


</td>
</tr>
</table>

Statistics

MEAN, MODE, MEDIAN AND RANGE

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulatives (counters or multi-link cubes). It may be useful for students to use different colours for the different values.	Students should draw the manipulatives as a diagram in their books.	Students should be comfortable with the formal definitions of mode, mean, median and range at this point.	Students should be able to find the mean, mode, median and range without needing a diagram. Higher attaining students should link to algebra during working.
Example(s)	**Example(s) - These should be seen in books**		
Find the mode, range, median and mean of 2, 5, 3, 7, 3.			
 Students should line up the manipulatives in order of size from the beginning. I.e.  **Mode** If different colours have been used this is the colour used for the most columns. If colours aren't used then students should use which height is the most common. **Range** Students should count the difference between the biggest value and the smallest value. **Mean** Students should aim to level out the manipulatives into a rectangle, removing one from the largest to the smallest. The mean is the height of each bar at the end of the process. It is important to highlight that the mean can be a decimal at this point.  **Median** Students should remove the smallest and biggest column and repeat. The median is the column that is remaining. Where there are two columns remaining students should find the mean of the two columns as above. 	Students should represent the numbers as bars using the squares in their books. This should be smallest to largest.  **Mode, Range and Mean** As Physical Stage. **Median** Students should cross off the numbers below the bars smallest then largest. Students should circle the median value. 	**Mode** Students at this point should know the mode is the most common value and should be able to determine this by inspection from the list of numbers. **Range** Students at this point should know the range is **Largest - Smallest** Students should clearly show this calculation, i.e. $7 - 2 = 5$ **Mean** At this point students should have identified the relationship that the total number of blocks/counters forms the area of the rectangle and the mean is the height of the rectangle. This should lead to the understanding that the mean is the total number divided by the amount of numbers.  **Median** Students at this point should know the median in the middle number. Students should be able to find the median without the aid of a diagram. Students should cross off the numbers, smallest then largest. Students should circle the median value. ~~2~~, ~~3~~ ③, ~~5~~, ~~7~~ ↑ Median = 3	**Mode** As semi-abstract stage. **Range** As semi-abstract stage. **Mean** Students should be confident with the definition and calculating the mean as below: $\frac{2+3+3+5+7}{5}$ Higher attaining students should be introduced to the formula written using sigma and notation, i.e. **Median** Students at this point should know the median in the middle number. Students should be able to state which number is the middle number using the formula Median Number = $\frac{n+1}{2}$ In this case: $\frac{n+1}{2} = \frac{5+1}{2} = 3$ So the median is the 3rd number in the list when the values are put in order.