The Levett School

Math's Calculations Policy

Policy agreed by Governors on:	3rd May 2022
Review date for Governors:	3rd May 2022
Allocated Group/Person to Review:	Hannah Buchanan
Agreed frequency of Review, by allocated person:	Every Year
Last Review date:	07.03.2022

Lower School, Melton Road, Sprotbrough, Doncaster, DN5 7SB
Upper School, Lansdowne Road, Intake, Doncaster, DN2 6QN

Recommended practice delivering a mastery approach

True mastery aims to develop all children’s mathematical understanding at the same pace. As much as possible, children should be accessing the samelearning. Differentiation should primarily be through support, scaffolding and deepening, not through task.

Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, ‘child-friendly’ terminology is used, this must be alongside the current terminology recommended by maths specialists. Using this will support children with their examinations and throughout secondaryschool.

Evidence repeatedly shows that mixed ability seating increases less confident pupils’ perception of mathematical capability, which impacts positively upon outcomes. While not a school policy, it is recommended to avoid ability groups. This presents a challenge in ensuring the more confident mathematicians arebeing extended. An extension tasks to deepen understanding is the most simplistic way around this.

Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to beused with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils.

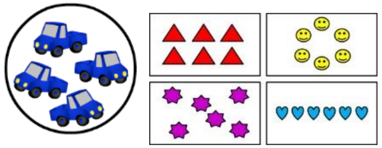
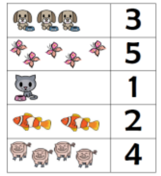
Used well, manipulatives can enable pupils to inquire themselves- becoming independent learners and thinkers. They can also provide a common language with which to communicate cognitive models for abstract ideas. Drury, H. (2015)	*Children aged seven to ten years old work in primarily concrete ways and that the abstract notions of mathematics may only be accessible to them through embodiment in practical resources. Jean Piaget's (1951)*	*Real things and structured images enables children to understand the abstract. The concrete and the images are a means for children to understand the symbolic so it’s important to move between all modes to allow children to make connections. Morgan, D. (2016)*

The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts.

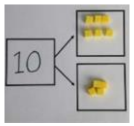
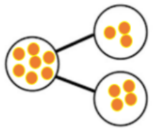
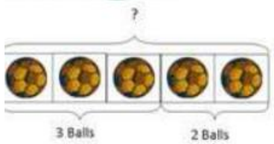
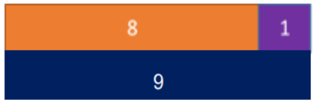
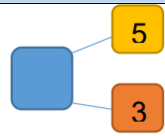
ADDITION

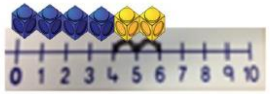
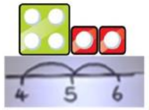
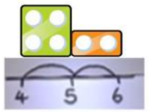
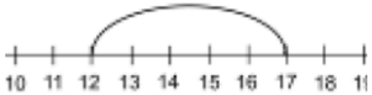
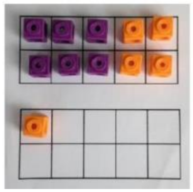
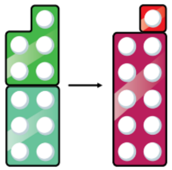
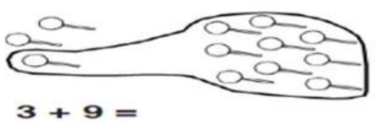
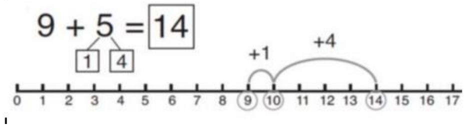
Vocabulary: sum, total, parts and wholes, plus, add, altogether, more, ‘is equal to’, ‘is the same as’

Stage 0 – pre-ARE (EYFS)

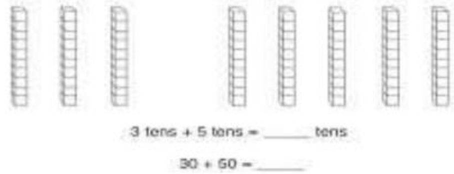
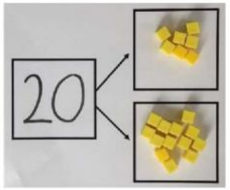
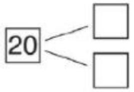
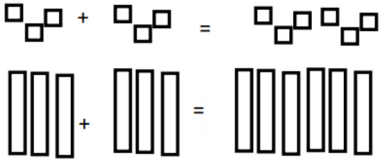
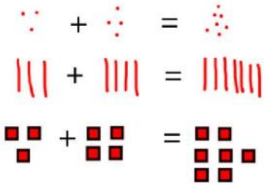
Method	Concrete	Pictorial	Abstract
Counting	Any item of the same things, e.g. pencils, pieces of pasta, shells, counters, cubes, cars, buttons,  	Pictures of the same items in different numbers and laid out differently. 	Relate the number of objects to the numeral. 

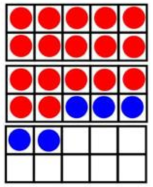
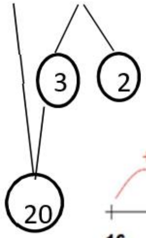
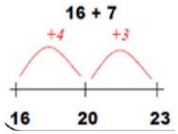
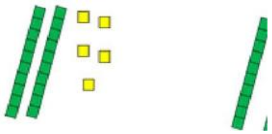
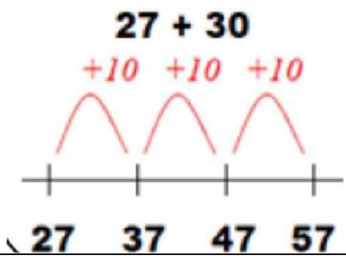
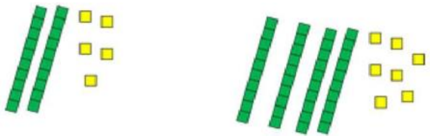
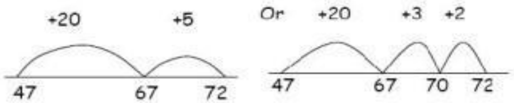
Stage 1 – Year 1

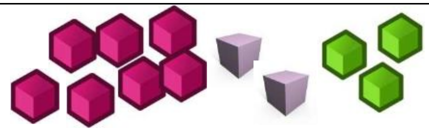
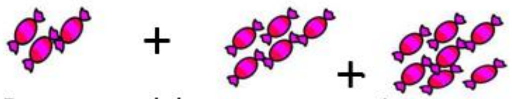
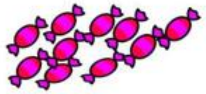
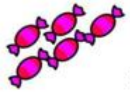
Method	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part whole model	 For 4 + 3, count out 4 items then 3 more and group them together to see what you have altogether. Represent in a bar / in a group   Other resources can be used instead of cubes (teddy’s, cars, shells, counters, etc).	Use pictures to add two numbers together as a group or in a bar.     	 Use the part – part – whole diagram as shown above to move into the abstract. 8 = 5 + 3 5 + 3 = 8 Include missing number questions to support varied fluency: 8 = ? + 3 5 + ? = 8

Starting at the bigger number and count on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.  Using number lines using cubes/ Numicon   	12 + 5 = 17  Start at the larger number from the sum on the number line and count on in 1s or jump to find the answer.	5 + 12 = 17 Place the number line in your head and count on the smaller number to find your answer. Variation of questions. With the number line in your head: • What is 2 more than 4? • What is the sum of 4 and 4? • What's the total of 4 and 2? • 4+2
'The Magic 10' **Regrouping to make 10** Makes the calculation easier. Essential for column addition later.	Regrouping 9 +3 into 10 + 2 before adding together  Start with the bigger number and use the smaller number to make 10 using ten frames or numicon: 6 + 5 = 11  	Children to draw the ten frame and counters/cubes. Use pictures or a number line. Regroup to partition the smaller number using the part – part- whole model to make 10.  	7 + 5 = ? 7 + 3 + 2 = ? If I have 7 how many of my 5 do I need to make 10? How many more do I still need to add on? Children to develop an understanding of equality 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □+ 4

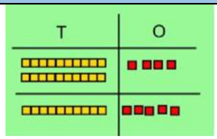
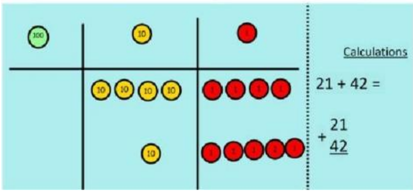
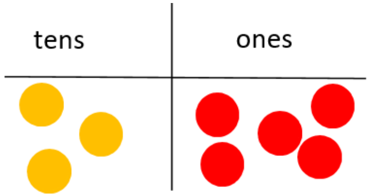
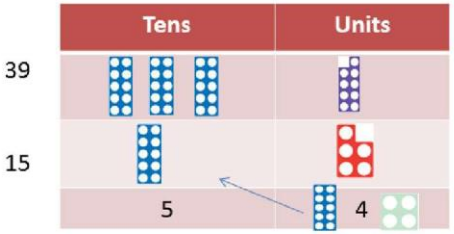
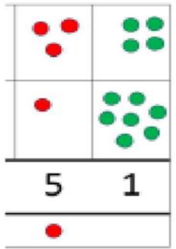
Stage 2 – Year 2

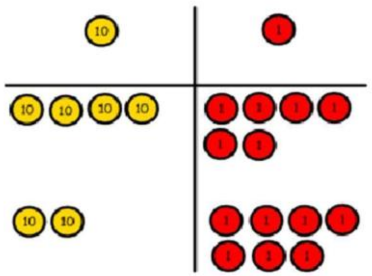
Method	Concrete	Pictorial	Abstract
Adding multiples of 10	50 = 30 + 20  Model using dienes and beadstrings.	Use representations of base ten.  	20 + 30 = 50 70 = 50 + 20 40 + ☐ = 60 Ensure all variations of sums layout is done.
Use known number facts Part, part whole	 Children explore ways of making numbers within 20.	 ☐ + ☐ = 20 20 – ☐ = ☐ ☐ + ☐ = 20 20 – ☐ = ☐	Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations. ☐ + 1 = 16 16 – 1 = ☐ 1 + ☐ = 16 16 – ☐ = 1
Using known facts		 Children draw representations of H, T and O.	3 + 4 = 7 leads to 30 + 40 = 70 leads to 300 + 400 = 700
Bar Model	 3 + 4 = 7	 7 + 3 = 10	23 | 25 ?

Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern.17 + 5 = 22 27 + 5 = 32	17 + 5 = 22 Use part part whole and number line to model.   	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17  Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.
Add a two digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change	 	27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2 digit numbers	 Model using dienes , place value counters and numicon	Use number line and bridge ten using part whole if necessary.  	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 =12 60 + 12 = 72 Lead into recording in column format, to reinforce place value and prepare childrenfor formal written methods with larger values.

Add three 1 digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation.  +  = 15	$\underbrace{4 + 7 + 6}_{10} = \boxed{10} + \boxed{7}$ $= \boxed{17}$ Combine the two numbers that make/bridge ten then add on the third.

Stage 3 – Year 3

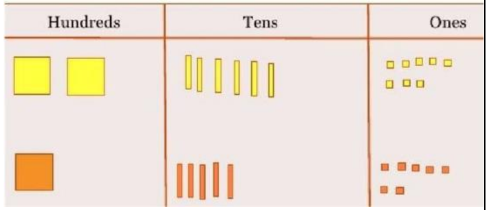
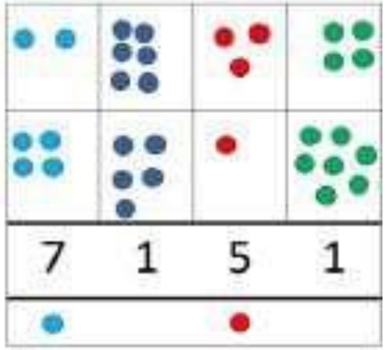
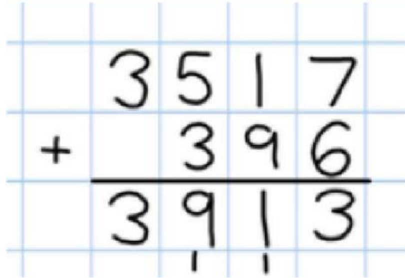
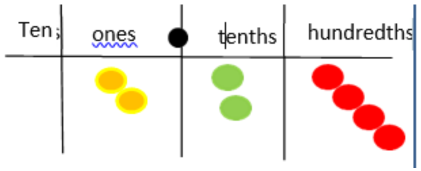
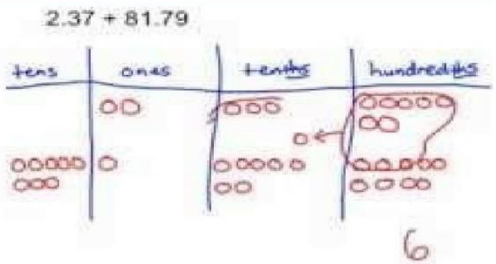
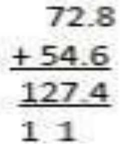
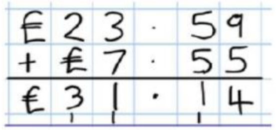
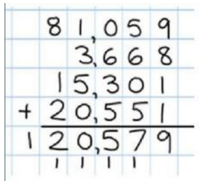
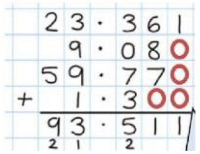
Method	Concrete	Pictorial	Abstract
Column Addition— no regrouping (friendly numbers) Add two or three 2 or3digit numbers.	 Dienes or numicon Add together the ones first, then thetens.   Move to using place value counters	Children move to drawing the counters using atens and one frame. 	$\begin{array}{r} 2\ 2\ 3 \\ +\ 1\ 1\ 4 \\ \hline 3\ 3\ 7 \end{array}$ Add the ones first, then the tens, thenthe hundreds.
Column Addition with regrouping .	 Exchange ten ones for a ten. Model using numicon and place value counters.	 $\begin{array}{r} 3\ 4 \\ +1\ 7 \end{array}$ Children can draw a representation of the grid to further support their understanding,carrying the ten **<u>underneath</u>** the line	$20 + 5$ $40 + 8$ $60 + 13 = 73$ Start by partitioning the numbers before formal column to showthe exchange. $\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ \hline 1\,1 \end{array}$

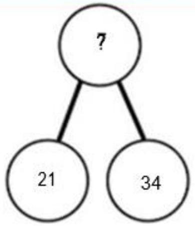
<table>
<tr><td></td><td>
46 + 27 = 73</td><td></td><td></td></tr>
<tr><td>Estimate the answers to questions and use inverse operations to check answers</td><td>
Estimating 98 + 17 = ?
100 + 20 = 120</td><td>Use number lines to illustrate estimation.

</td><td>Building up known facts and using them to illustrate the inverse and to check answers.

98 + 18 = 116 116 – 18 = 98

18 + 98 = 116 116 – 98 = 18</td></tr>
<tr><td colspan="4">Stage 4 – Year 4-6</td></tr>
<tr><td>Method</td><td>Concrete</td><td>Pictorial</td><td>Abstract</td></tr>
<tr><td>Years 4 – 6
Estimate and use inverse operations to check answers to a calculation</td><td colspan="3">As per Year 3</td></tr>
</table>

Y4—add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundredand ten hundreds for a thousand.  	 Draw representations using place value grid.	 Continue from previous work to carryhundreds as well as tens. Relate to money and measures.
Y5—add numbers with	As year 4  Introduce decimal place value countersand model exchange for addition.	 	 
Y6—add several numbers of increasing complexity, inc adding money, measureand decimals with different numbers of decimal points.	As Y5	As Y5	Insert zeros for place holders.  
Conceptual variation; different ways to ask pupils to solve 21+34			



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

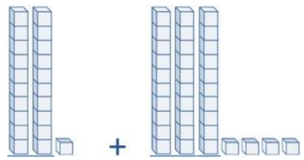
21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 =

$\square$ = 21 + 34

Calculate the sum of twenty-one and thirty-four.

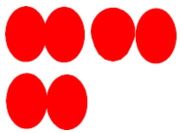
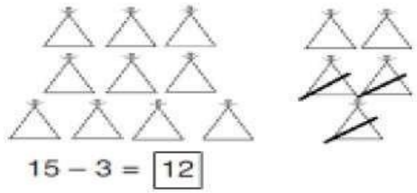
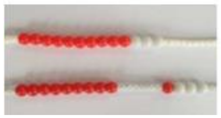
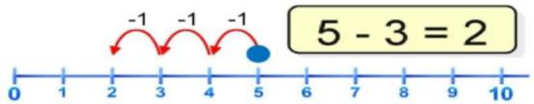
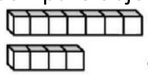
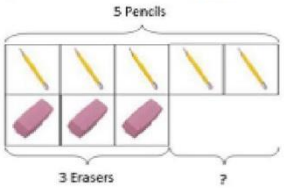
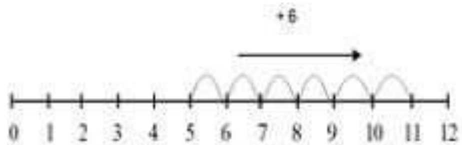


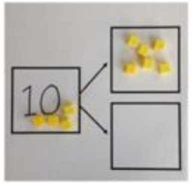
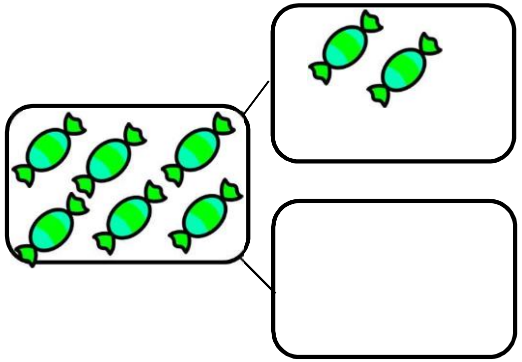
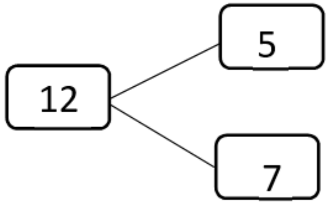
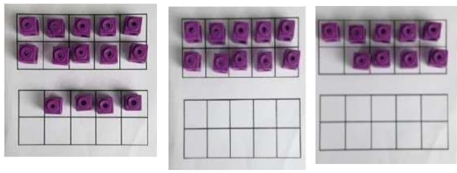
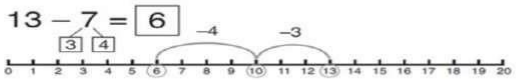
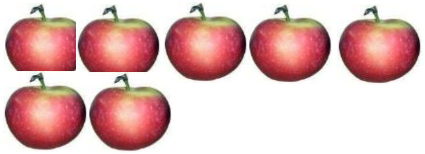
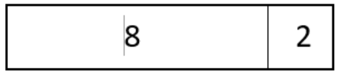
Missing digit problems:

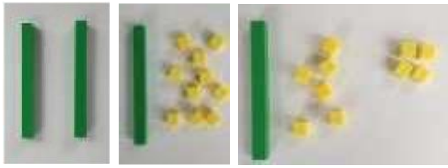
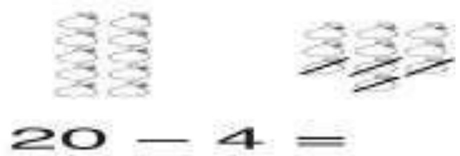
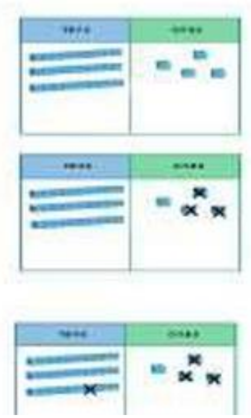
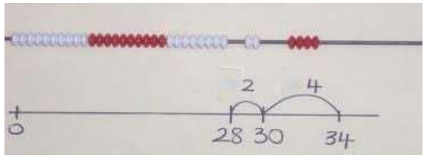
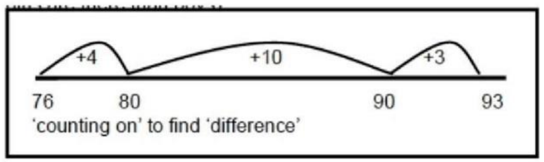
SUBTRACTION

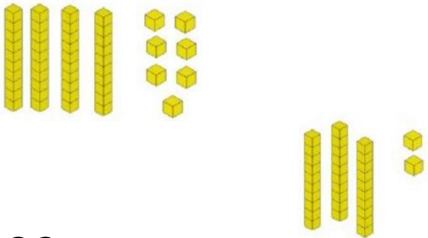
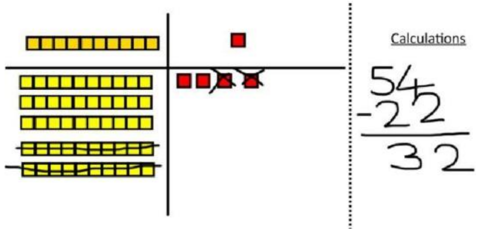
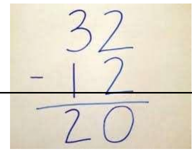
Vocabulary: take away, less than, the difference, subtract, minus, fewer, decrease

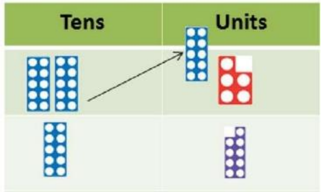
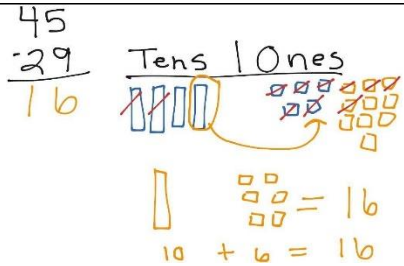
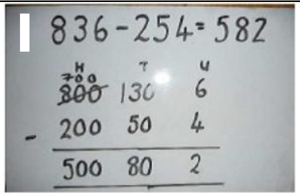
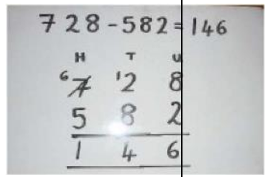
Stage 1 – Year 1

Method	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etcto show how objects can be taken away. 4—2 = 2  6—4 = 2 	Cross out drawn objects to show what hasbeen taken away.  	7—4 = 3 16—9 = 7
Counting back	  Move objects away from the group, counting backwards.  Move the beads along the bead string as you countbackwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. Whatnumber are you at?
Find the Difference	Compare objects and amounts  7 'Seven is 3 more than four' 4 'I am 2 years older than my sister'  	Count on using a number line to find the difference.  	Hannah has12 sweets and her sister has 5. How many more does Hannah have than hersister.?

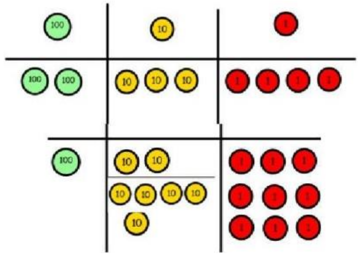
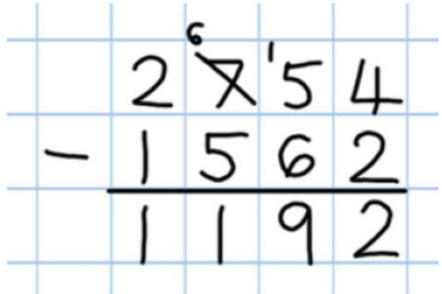
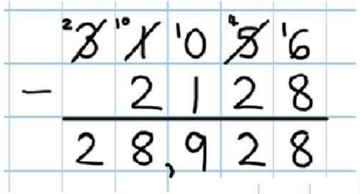
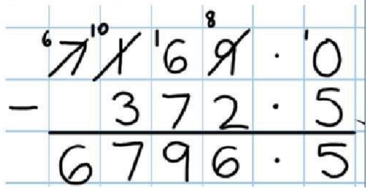
	Lay objects to represent bar model.		
Represent and use number bonds and related subtraction facts within 20 Include subtracting zero Part Part Whole model	 Link to addition. Use PPW model to modelthe inverse. If 10 is the whole and 6 is one of the arts,what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within thepart whole model.  Include missing number problems: 12 - ? = 5 7 = 12 - ?
Make 10 using a ten frame	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away sothat you have taken 5.	 13—7 Jump back 3 first, then another 4. Use tenas the stopping point.	16—8 How many do we take off first to get to 10?How many left to take off?
Bar model Including the inverse operations.	 5—2 = 3		 10 = 8 + 2 10 = 2 + 8 10—2 = 8

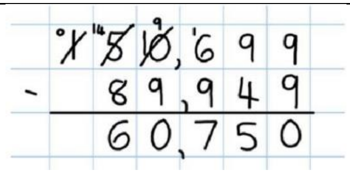
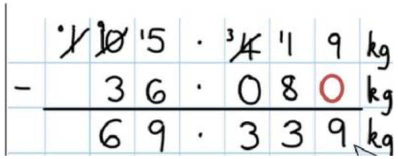
			10—8 = 2
Stage 2 – Year 2			
Method	Concrete	Pictorial	Abstract
Regroup a ten intoten ones	 Use a PV chart to show how to change a ten into ten ones, use the term ‘take andmake’	 	20—4 = 16
Partitioning to subtract without regrouping. *‘Friendly numbers’*	34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes andcross off.  43—21 = 22	43—21 = 22
Make ten strategy *Progression should be crossing one ten, crossingmore than one ten,*	 34—28	 Use a number line to count on to next tenand	93—76 = 17

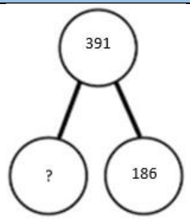
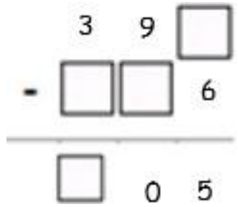
crossing the hundreds.	Use a bead bar or bead strings to modelcounting to next ten and the rest.	then the rest.	
Stage 3 – Year 3			
Method	Concrete	Pictorial	Abstract
Subtract numbers mentally, including: three digit number + ones three digit number + tens three digit number + hundreds			Vary the position of the answer and question. Expose children to missing number questions and vary the missing part of the calculation. 678 = ? - 1 688 – 10 = ? 678 = ? – 100
Column subtraction without regrouping (friendly numbers)	 47—32 Use base 10 or Numicon to model	 Draw representations to supportunderstanding	47 − 24 = 23 40 + 7 − 20 + 4 20 + 3 Intermediate step maybe needed to lead to clear subtraction understanding. 

Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take andmake' for exchange.	 Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns  Then move to formal method.

Stage 4 – Years 4-6

Method	Concrete	Pictorial	Abstract
Subtracting tens andones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon,base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with*	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for placeholder 

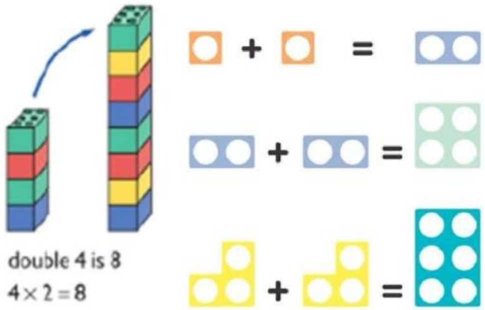
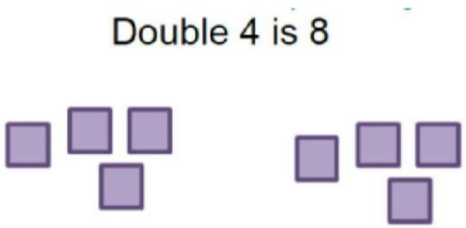
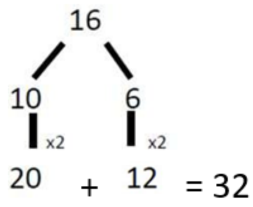
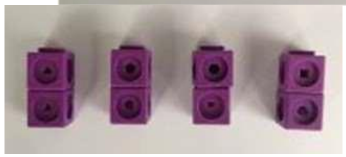
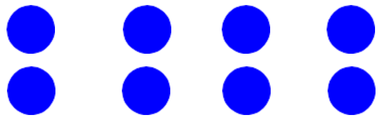
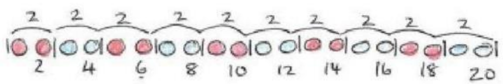
decimal values, including mixturesof integers and decimals and aligning the decimal *Up to 3 decimal places*			
Year 6—Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).	As Year 4	Children to draw pv counters and show their exchange—see Y3	 

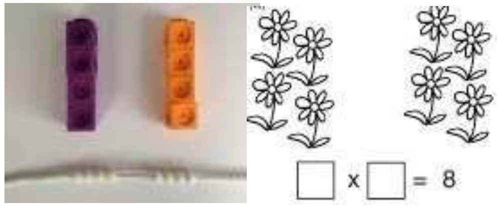
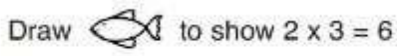
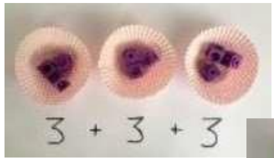
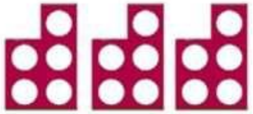
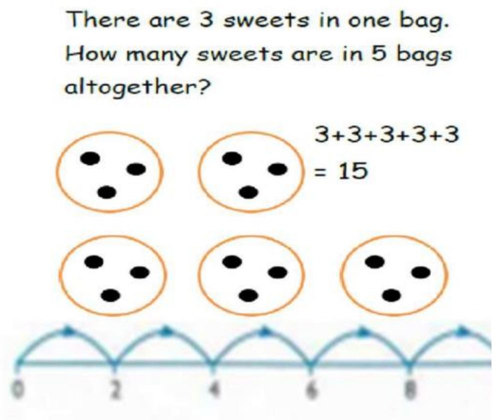
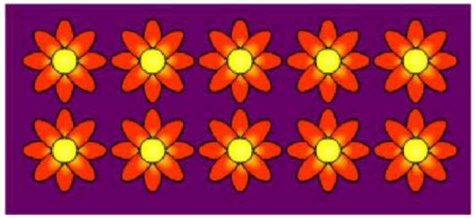
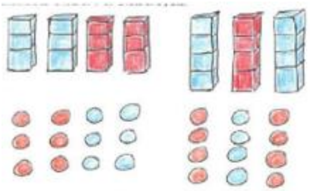
Conceptual variation; different ways to ask pupils to solve 391- 186			
 391 186 | ?	Raj spent £391, Timmy spent £186. How much more did Raj spend? Calculate the difference between 391 and 186.	☐ = 391 – 186 391 -186 ____ What is 186 less than 391?	Missing digit calculations 

MULTIPLICATION

Vocabulary: double, times, multiplied by, the product of, groups of, lots of, equal groups

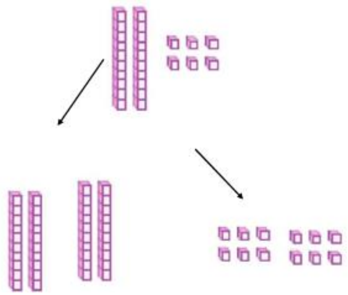
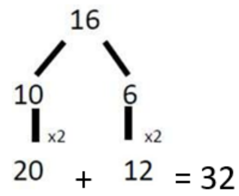
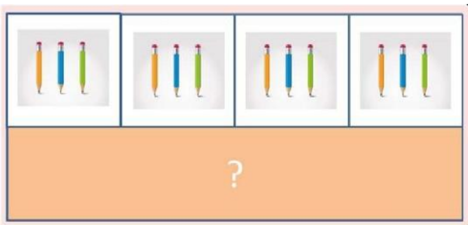
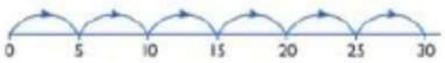
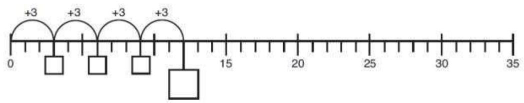
Stage 1 – Year 1

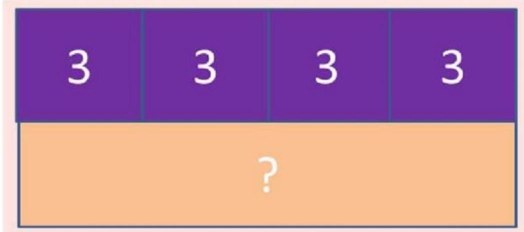
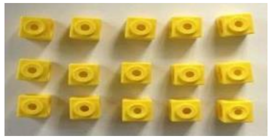
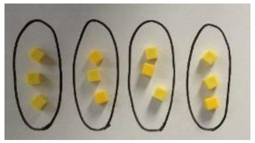
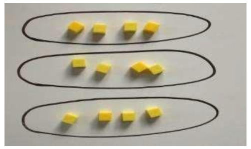
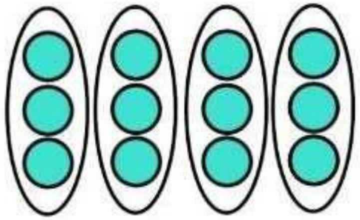
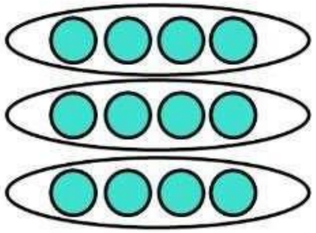
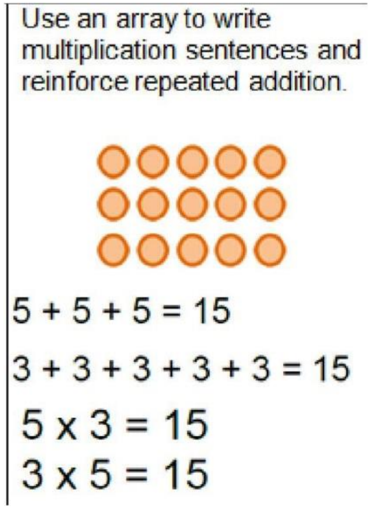
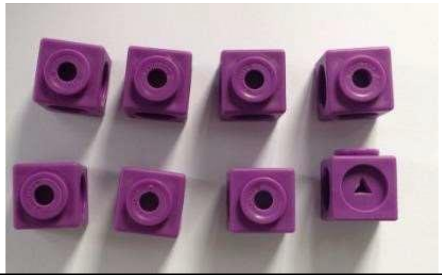
Method	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers 	Partition a number and then double each partbefore recombining it back together. 
Counting in multiples (2s, 5s, 10s)	Count the groups as children are skip counting, children may use their fingers as they are skip counting.  	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples ofnumbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

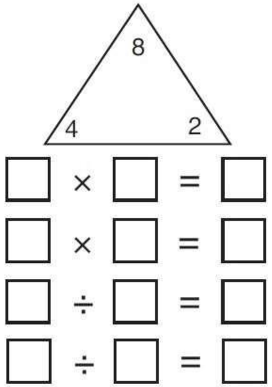
Making equal groups and counting the total	 Use manipulatives to create equal groups.	 Draw and make representations	2 x 4 = 8
Repeated addition	   Use different objects to addequal groups	Use pictorial including number lines to solve problems 	Write addition sentences to describe objectsand pictures. 
Understanding arrays	Use objects laid out in arrays to find theanswers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show  Understanding,	3 x 2 = 6 2 x 5 = 10

Stage 2 – Year 2

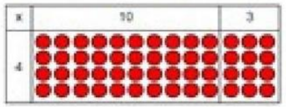
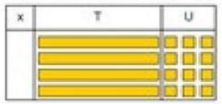
Children should be able to recall and sue multiplication and division facts for the 2, 5 and 10 times tables.

Method	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes andPV counters.  40 + 12 = 52	Draw pictures and representations toshow how to double numbers	Partition a number and then double eachpart before recombining it back together. 
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children areskip counting, children may use their fingers as they are skip counting. Use bar models.   5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.    	Count in multiples of a number aloud. Write sequences with multiples ofnumbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 $4 \times 3 = \square$

			
Multiplication is commutative	Create arrays usingcounters and cubes and Numicon.   Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4

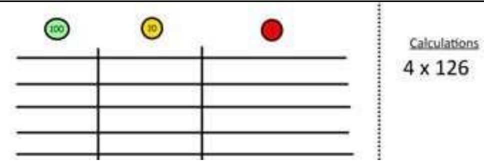
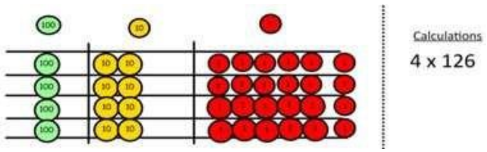
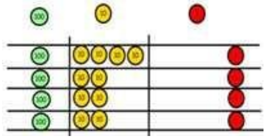
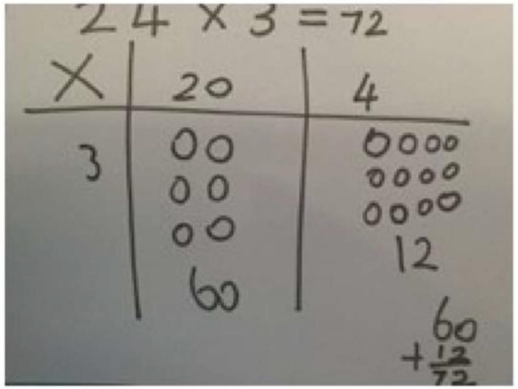
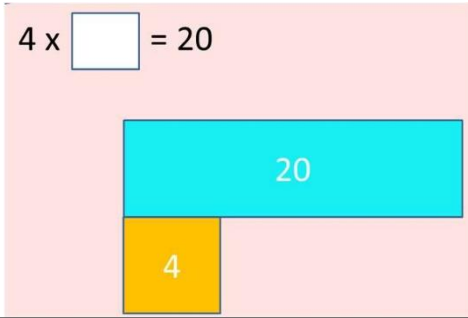
<table>
<tr><td></td><td></td><td></td><td>4 = 8÷ 2

Show all 8 related fact family sentences.</td></tr>
<tr><td colspan="4">Stage 3 – Year 3
Children should be able to recall and sue multiplication and division facts for the 2, 5 and 10 times tables.</td></tr>
<tr><td>Method</td><td>Concrete</td><td>Pictorial</td><td>Abstract</td></tr>
<tr><td>Grid method, progressing to theformal method

Multiply 2 digit numbers by 1 digit numbers</td><td>Show the links with arrays to first introduce the grid method.

4 rows of 10
4 rows of 3
Move onto base ten to move towards a morecompact method.

4 rows of 13
Move on to place value counters to show how we are finding groups of a number. We aremultiplying by 4 so we need 4 rows</td><td>Children can represent their work with place value counters in a way that they understand.

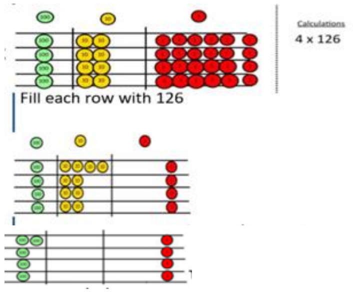
They can draw the counters using colours to show different amounts or just use the circles inthe different columns to show their thinking as shown below.</td><td>Start with multiplying by one digit numbers and showing the clear addition alongside thegrid.
<table><tr><td>×</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table>
210 + 35 = 245

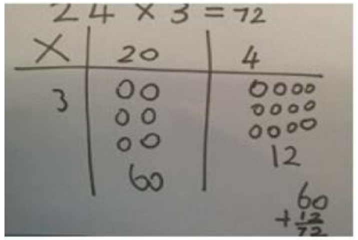
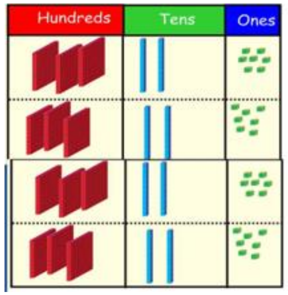
Move forward to the formal written method:
3 5
X 7
2 4 5</td></tr>
</table>

	  Fill each row with 126. Add up each column, starting with the onesmaking any exchanges needed Then you have your answer. 	 	3
Solve problems, including missing number problems, integer scaling problems,			Three times as high, eight times as long ? x 5 = 20 20 ÷ ? = 5 3 hats and 4 coats, how many differentoutfits?

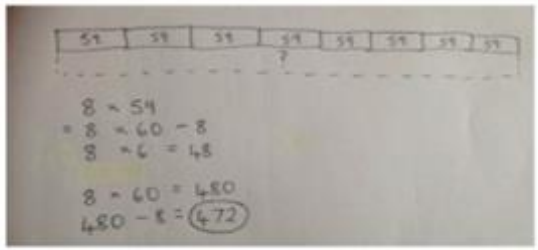
Stage 4 – Years 4-6

Method	Concrete	Pictorial	Abstract
Grid method recap from Year 3 for 2 digits x 1 digit	Use place counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show	Start by multiplying by one digit numbers and showing the clear addition alongside the grid.

<table>
<tr>
<td>Move to multiplying 3 digit numbers by 1 digit. (Year 4 expectation).</td>
<td>

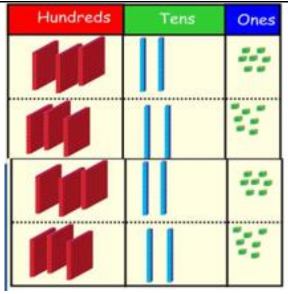
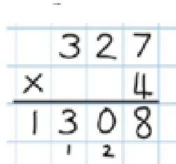
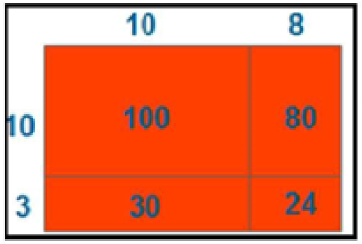
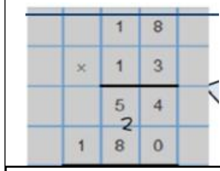
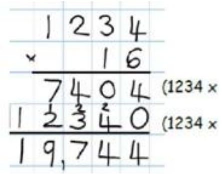
Add up each column making any exchanges as needed.</td>
<td>different amounts or just use the circles in the different columns to show their thinking as shown below.

</td>
<td><table><tr><td>×</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table>
210 + 35 = 245</td>
</tr>
<tr>
<td>Column multiplication</td>
<td>Children can continue to be supported by place value counters at this stage of multiplication. This is initially done where there is no regrouping.
321 x 2 = 642


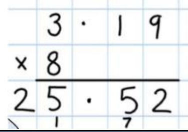
It is important at this stage that they always multiply the ones first.

The corresponding long multiplication is modelled alongside.</td>
<td><table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>
The grid method may be used to show how this relates to a formal written method.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</td>
<td>327
x 4
28
80
1200
1308

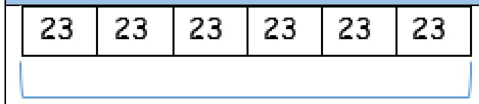
This may lead to a compact method.

<table><tr><td></td><td>3</td><td>2</td><td>7</td></tr><tr><td>×</td><td></td><td></td><td>4</td></tr><tr><td>1</td><td>3</td><td>0</td><td>8</td></tr><tr><td></td><td>1</td><td>2</td><td></td></tr></table></td>
</tr>
</table>

Column Multiplication for3 and 4 digits x 1 digit.	 It is important at this stage that they always Multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642	<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>	327 x 4 28 80 1200 1308 
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	 Continue to use bar modelling to support problem solving	  18 x 3 on thefirst row (8 x 3 =24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first
Multiplying decimalsup to 2 decimal places by a single digit.			Remind children that the single digit belongsin the units column. Line up the decimal points in the question and the answer.

Conceptual variation; different ways to ask pupils to solve

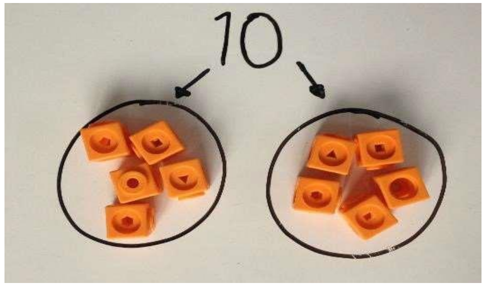
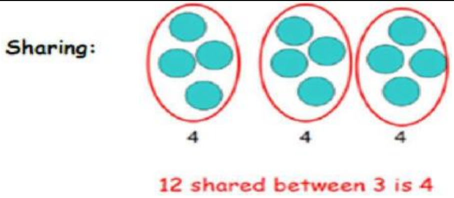
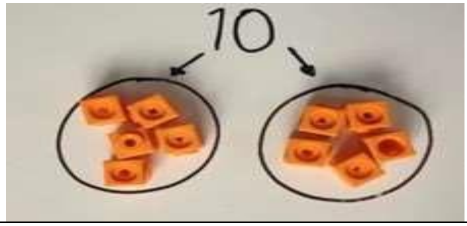
	Mai had to swim 23 lengths, 6 timesa week. How many lengths did she swim inone week? With the counters, prove that 6 x 23 = 138	Find the product of 6 and 23 6 × 23 = $\square$ = 6 × 23 $\begin{array}{r} 6 \\ \times\ 23 \\ \hline \end{array}$ $\begin{array}{r} 23 \\ \times\ 6 \\ \hline \end{array}$	What is the calculation?What is the product? 

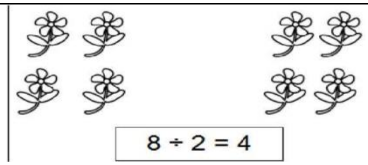
DIVISION

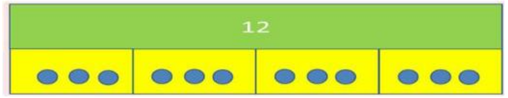
Vocabulary: share, group, divide, divided by, half

Stage 1 – Year 1

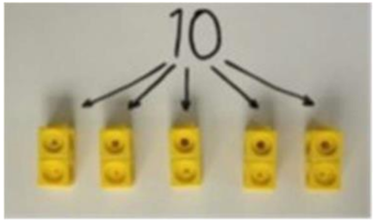
Method	Concrete	Pictorial	Abstract
Division as sharing *Use Gordon ITPs for modelling*		Children use pictures or shapes to share quanti ties.  8 shared between 2 is 4	12 shared between 3 is 4

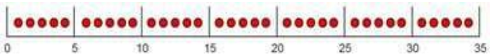
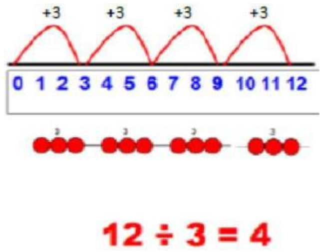
	   I have 10 cubes, can you share them equally in2 groups?	 	
Division as sharing	 	Children use pictures or shapes to share quantities.	$12 \div 3 = 4$

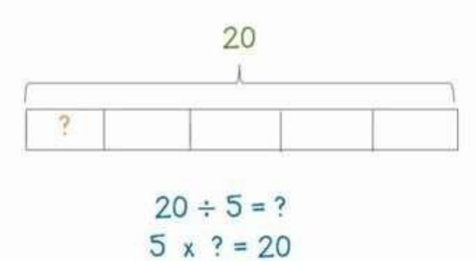
<table>
<tr><td></td><td>I have 10 cubes, can you share them equally in2 groups?</td><td>

Children use bar modelling to show and support understanding.


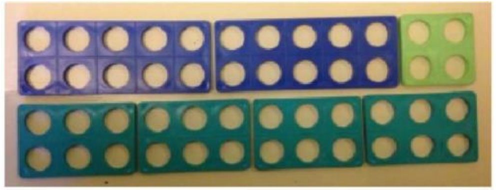
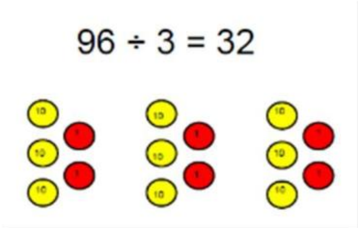
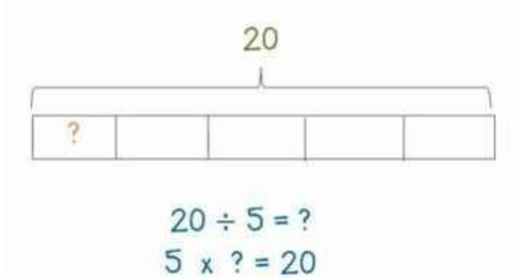
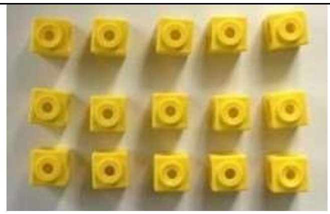
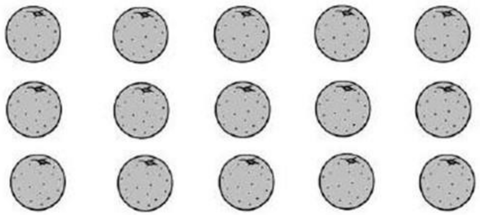
12 ÷ 4 = 3</td><td></td></tr>
<tr><td>Division as grouping</td><td>Divide quantities into equal groups.

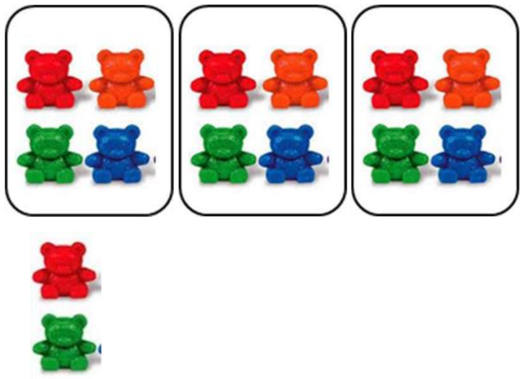
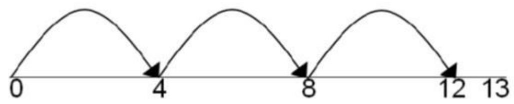
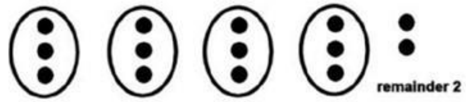
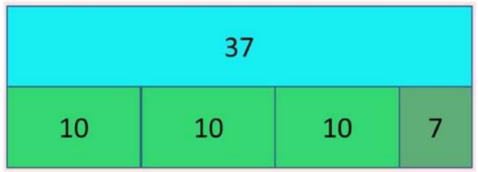
Use cubes, counters, objects or place value counters to aid understanding.


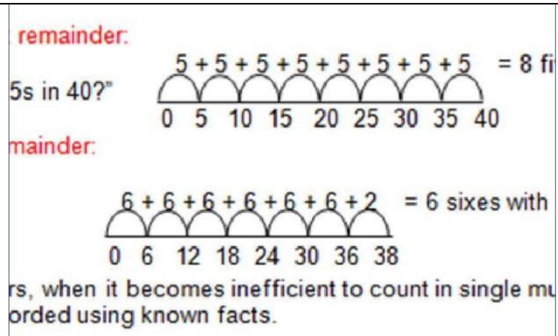
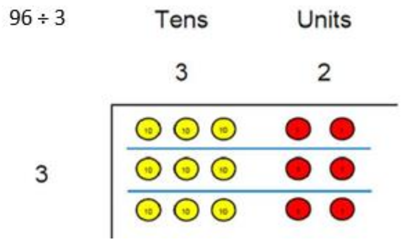
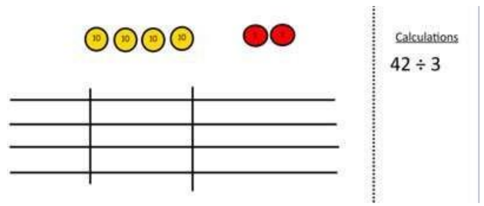
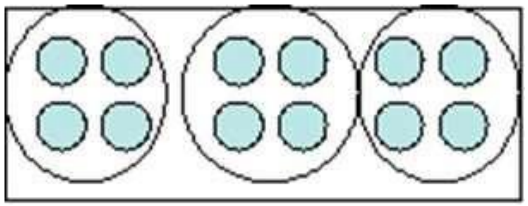
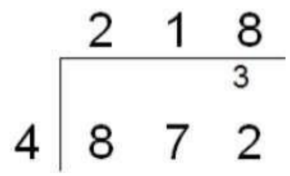
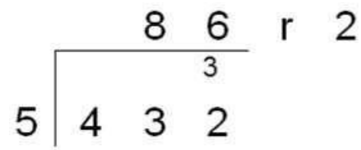
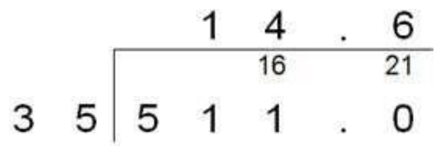

</td><td>Use number lines for grouping


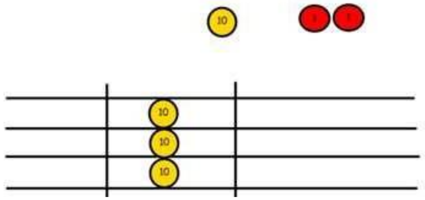
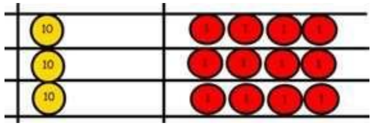
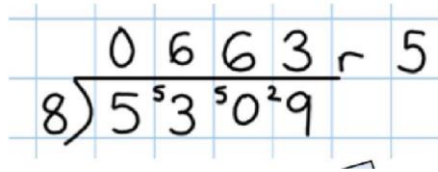
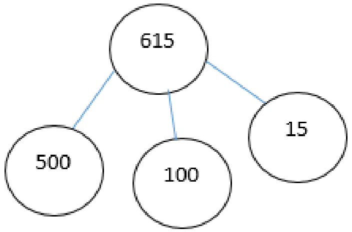
Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.

</td><td>28 ÷ 7 = 4

Divide 28 into 7 groups. How many are ineach group?</td></tr>
<tr><td colspan="4">Stage 2 – Year 2</td></tr>
</table>

Method	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4  	Continue to use bar modelling to aid solving division problems.  	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 =15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Stage 3 – Year 3 (Greater Depth Year 2)			
Method	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups andsee how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders. 	Complete written divisions and show theremainder using r. 

			
Stage 4 – Year 4-6			
Method	**Concrete**	**Pictorial**	**Abstract**
Divide at least 3 digitnumbers by 1 digit. Short Division	  Use place value counters to divide using the bus stop method alongside 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally withno remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide thetotal accurately. 

<table>
<tr>
<td></td>
<td>
We exchange this ten for ten ones and then share the ones equally among the groups.

We look how much in 1 group so the answeris 14.</td>
<td></td>
<td>
</td>
</tr>
<tr>
<td colspan="4">Conceptual variation; different ways to ask children to solve 615 ÷ 5</td>
</tr>
<tr>
<td>Using the part whole model below, howcan you divide 615 by 5 without using short division?

</td>
<td>have £615 and share it equally between 5 bank accounts. How muchwill be in each account?

615 pupils need to be put into 5 groups. How many will be in eachgroup?</td>
<td>$5\overline{)615}$

615 ÷ 5 =

$\boxed{}$ = 615 ÷ 5</td>
<td>What is the calculation?What is the answer?

</td>
</tr>
</table>

Long Division

Step 1—a remainder in the ones

h t o

0 4 1 R1

4) 1 6 5

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

th h t o

0 4 0 0 R7

8) 3 2 0 7

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Step 1 continued…..

$$\begin{array}{r} \text{h t o} \\ 0\ 6\ 1 \\ 4\,\overline{)\,2\ 4\ 7} \\ -\ 4 \\ \hline 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

$$\begin{array}{r} \text{th h t o} \\ 0\ 4\ 0\ 2 \\ 4\,\overline{)\,1\ 6\ 0\ 9} \\ -\ 8 \\ \hline 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Step 2 – a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2) 5 8 Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	t o 2 2) 5 8 - 4 1 To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	t o 2 9 2) 5 8 - 4 ↓ 1 8 Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2) 5 8 - 4 1 8 Divide 2 into 18. Place 9 into the quotient.	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0 The division is over since there are no more digits in the dividend. The quotient is 29.

Step 3 – a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2) 2 7 8 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2) 2 7 8 - 2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 1 8 2) 2 7 8 - 2 ↓ 0 7 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2) 2 7 8 - 2 0 7 Divide 2 into 7. Place 3 into the quotient.	h t o 1 3 2) 2 7 8 - 2 0 7 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 1 3 2) 2 7 8 - 2 0 7 - 6 1 8 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2) 2 7 8 - 2 0 7 - 6 1 8 Divide 2 into 18. Place 9 into the quotient.	h t o 1 3 9 2) 2 7 8 - 2 0 7 - 6 1 8 - 1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the	h t o 1 3 9 2) 2 7 8 - 2 0 7 - 6 1 8 - 1 8 0 There are no more digits to drop down. The quotient is 139.

Conceptual variation; different ways to ask pupils to solve 615 ÷ 5